Important Concepts for Stat 61

Calculus and Algebra Exercises:

1. Find the derivative with respect to $x$ of $f(x) = xe^{-x}$.
2. Find the derivative with respect to $x$ of $f(x) = ax$ (assume $a > 0$).
3. Find the antiderivative of $f(x) = \log(x)$ (i.e., evaluate the indefinite integral $\int \log(x)\,dx$).
4. Find the antiderivative of $f(x) = xe^{-x}$.
5. Show that $\int_{-\infty}^{\infty} e^{-x^2/2}\,dx = \sqrt{2\pi}$. Hint: Show that the square of the integral equals $2\pi$. Write it as a double integral for variables $x$ and $y$ and convert to polar coordinates.
6. Show that $\sum_{n=0}^{\infty} \frac{a^n}{n!} = e^a$ (Hint: it’s a Taylor series expansion about $a = 0$).
7. Show that $\lim_{n \to \infty} (1 + a/n)^n = e^a$ (Hint: use the binomial expansion).
8. For $x$ a fixed, positive integer, show that $\lim_{n \to \infty} \frac{n!/(n-x)!}{n^n} = 1$.
9. Show that, for $a > 0$,
   \[ \int_0^{\infty} x^a e^{-x} \,dx = a \int_0^{\infty} x^{a-1} e^{-x} \,dx. \]
10. Find the value of $\mu$ (in terms of the $x_i$’s) that maximizes the function
   \[ L(\mu) = \exp \left[ -\frac{1}{2} \sum_{i=1}^{n} (x_i - \mu)^2 / 2 \right]. \]
11. Show that
   \[ \sum_{i=1}^{n} (x_i - \mu)^2 = n(\bar{x} - \mu)^2 + \sum_{i=1}^{n} (x_i - \bar{x})^2, \quad \text{where} \quad \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i. \]
12. Suppose $f(x, y) = 4e^{-2y}$ for $0 < x < y$, and $f(x, y) = 0$ otherwise. Show that the total volume under $f$ is $1$. Also find the volume over the region where $x > 1$.

Elementary Statistics Concepts:
Elementary statistics (e.g., AP Stat, Stat 11 or Econ 31) is not required for Stat 61, but it is “strongly recommended”.

1. What is the difference between an experiment and an observational study, and why is the distinction important?
2. What is the difference between a Simple Random Sample (SRS) and a Stratified Random Sample?
3. What is a test of hypotheses, and what is the definition of the P-value for a test?
4. What does it mean to say a result is statistically significant? What does it mean to say a result is not statistically significant?
5. What does it mean if the margin of error for a 95% confidence interval is 0.03?
6. What is meant by the sampling distribution for a statistic?