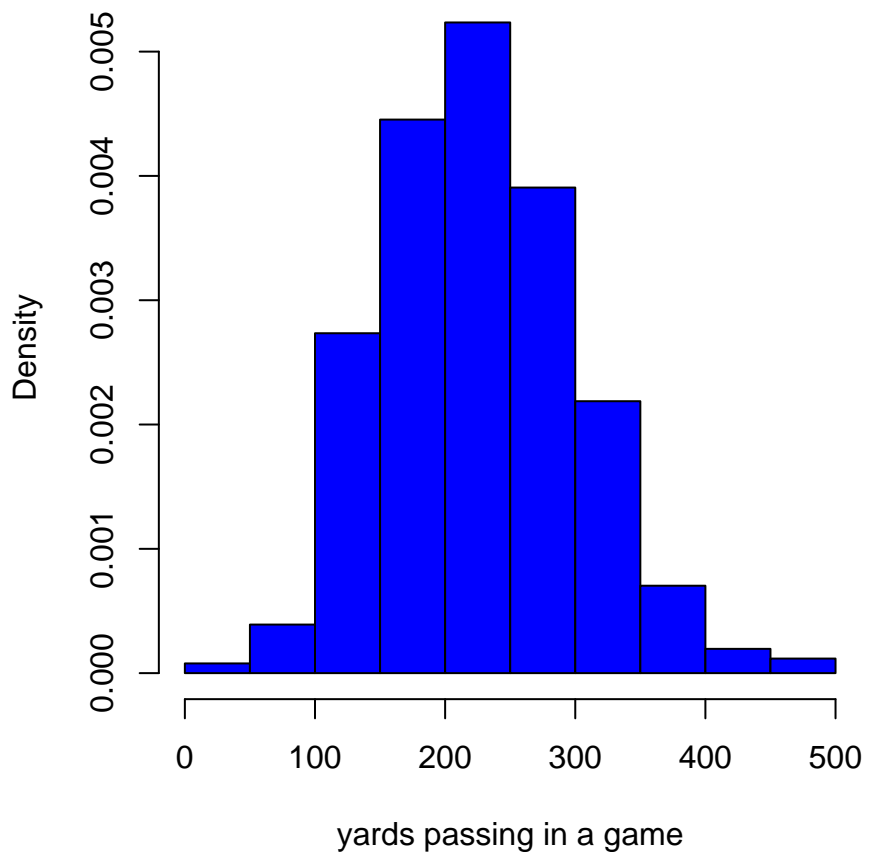


A Bivariate Normal Model for NFL Passing Yardage

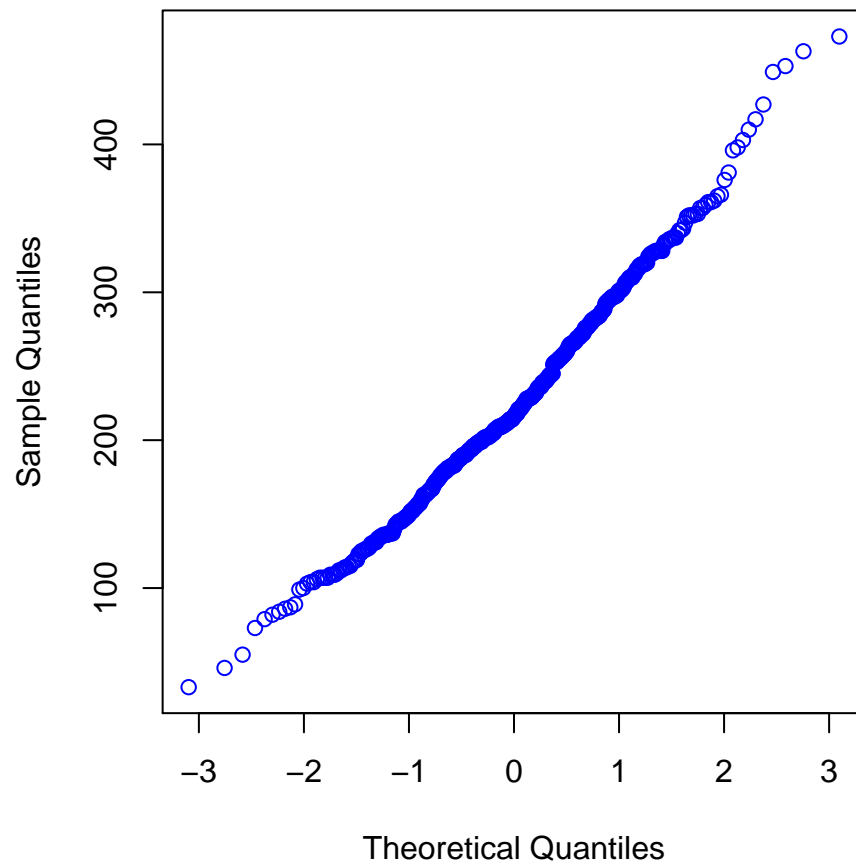
Phil Everson
Swarthmore College, PA
August, 2003

2002 NFL Passing Yards

256 total games; 512 yardages

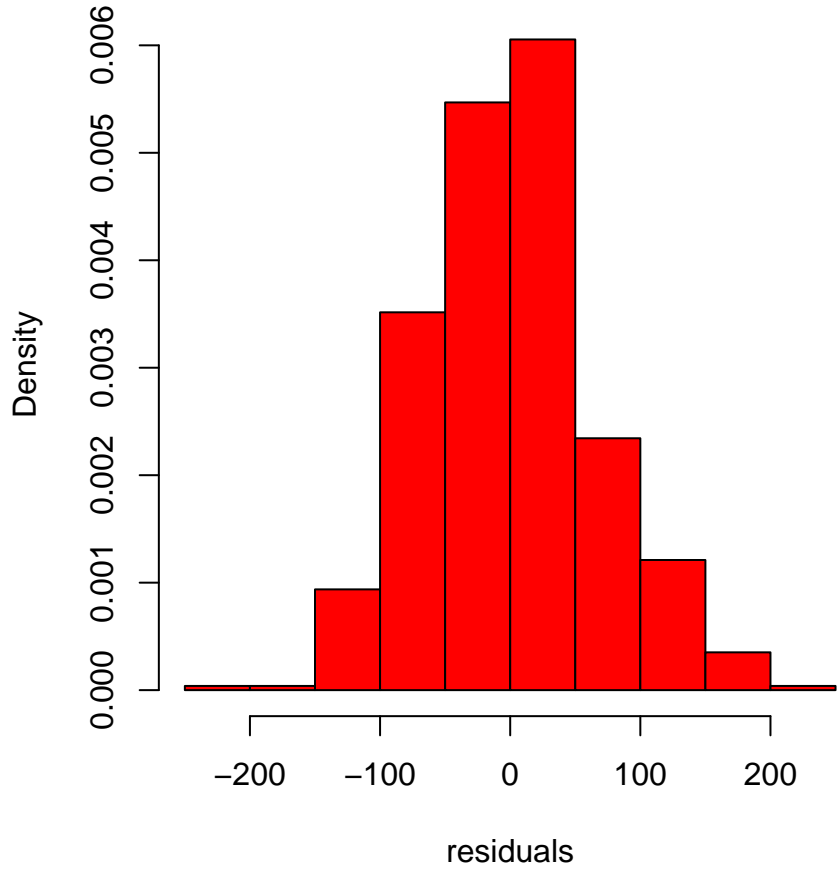


Normal Q-Q Plot

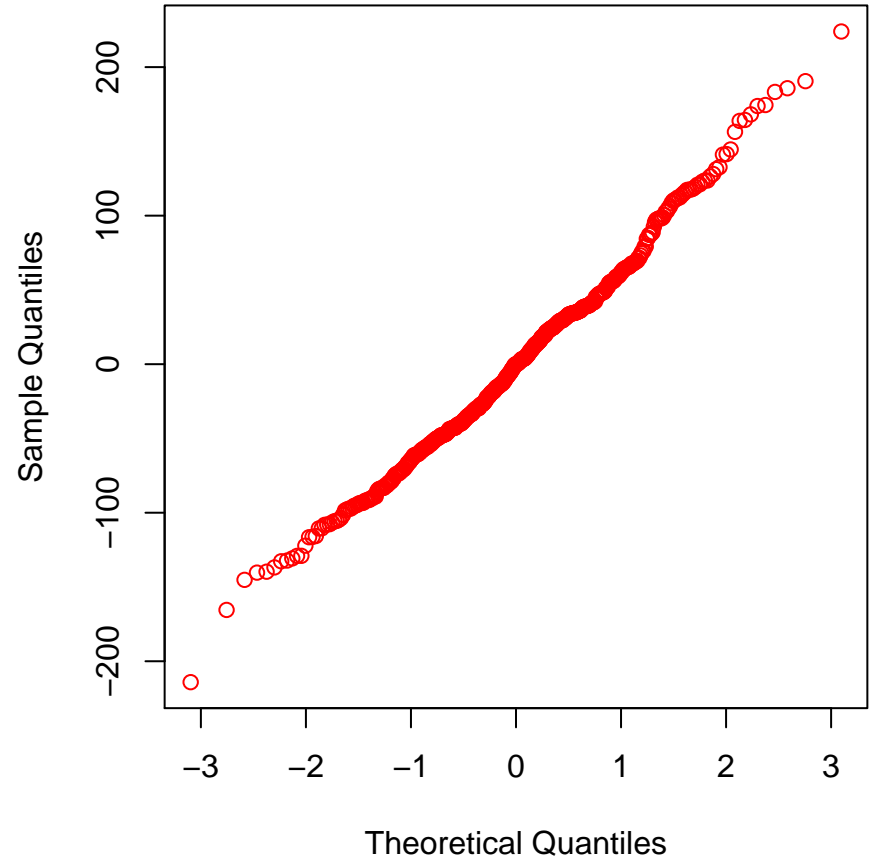


Deviations from Team Averages

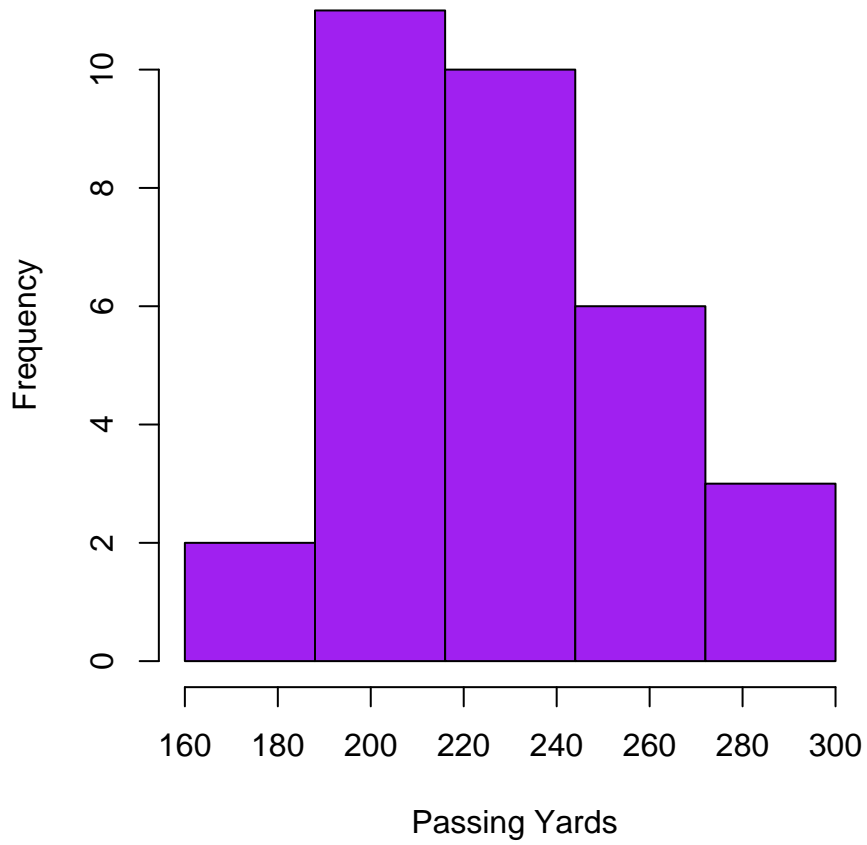
RMSE = 68 yards



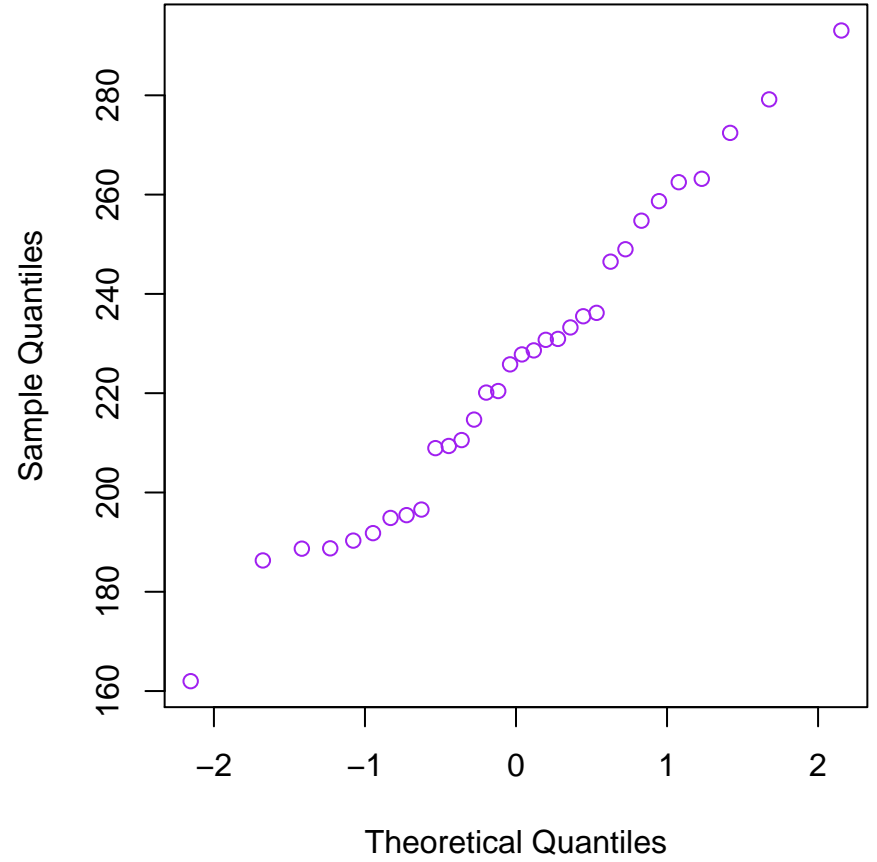
Normal Q-Q Plot



Team Averages



Normal Q-Q Plot



Univariate Model for Passing Yards

Y_{ij} = passing yards for team i in its j th game.

Y_i = **average** yards-per-game for team i in n_i games.

$$= \frac{1}{n_i} \sum_{j=1}^{n_i} Y_{ij}, \quad i = 1, \dots, 32 \text{ teams.}$$

$$\hat{\sigma}^2 = \frac{1}{2N - 32} \sum_{i=1}^{32} \sum_{j=1}^{n_i} (Y_{ij} - Y_i)^2 = \text{MSE.}$$

Season totals:

Average = 225 yards. RMSE = 68 yards.

Univariate Normal Hierarchical Model

Level - 1

$$Y_i | \theta_i, \sigma \sim N(\theta_i, V_i), \quad V_i = \sigma^2/n_i, \quad i = 1, \dots, 32$$

$$\hat{\sigma}^2 | \sigma \sim \text{Gamma}\left(\frac{2N - 32}{2}, \frac{2N - 32}{2\sigma^2}\right)$$

Level - 2

$$\theta_i | \beta, A \sim N(\beta, A)$$

Conditional Posterior Distribution:

$$\theta_i | Y_i, \beta, A, \sigma \sim N(B_i\beta + (1 - B_i)Y_i, (1 - B_i)V_i)$$

$$B_i = \frac{V_i}{V_i + A}, \quad V_i = \frac{\sigma^2}{n_i}$$

Bivariate Team Parameters

$$\theta_i = \begin{pmatrix} \theta_i \text{ offense} \\ \theta_i \text{ defense} \end{pmatrix} \sim N_2(\beta, \mathbf{A}), \quad i = 1, \dots, 32 \text{ teams.}$$

Model for data:

If team i plays its j th game at home against team i' ,

$$Y_{ij} = \begin{pmatrix} \text{team } i \\ \text{pass yards} \\ \text{team } i' \\ \text{pass yards} \end{pmatrix} \sim N_2(\mu_{ij}, \mathbf{\Sigma})$$

$$\mu_{ij} = \begin{pmatrix} \theta_i \text{ offense} + \theta_{i'} \text{ defense} \\ \theta_i \text{ defense} + \theta_{i'} \text{ offense} \end{pmatrix}$$

Conditional Estimates

$$\hat{\theta}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} (Y_{ij} - \delta_{ij}) \mid \theta, \mathbf{V}_i \sim N_2(\theta_i, \mathbf{V}_i), \quad i = 1, \dots, 32$$

$$\theta_i = \begin{pmatrix} \theta_{i \text{ offense}} \\ \theta_{i \text{ defense}} \end{pmatrix}, \quad \delta_{ij} = \begin{pmatrix} \theta_{i'_j \text{ defense}} \\ \theta_{i'_j \text{ offense}} \end{pmatrix},$$

where i'_j is the index for the j th opponent of team i .

$$\hat{\Sigma} = \frac{1}{2N} \sum_{i=1}^{32} \sum_{j=1}^{n_i} (Y_{ij} - \mu_{ij})(Y_{ij} - \mu_{ij})^T \sim \text{Wishart}_2(N, N\Sigma^{-1})$$

$$\mu_{ij} = \begin{pmatrix} \theta_{i \text{ offense}} + \theta_{i'_j \text{ defense}} \\ \theta_{i \text{ defense}} + \theta_{i'_j \text{ offense}} \end{pmatrix}$$

$$\hat{\mathbf{A}} = \frac{1}{32} \sum_{i=1}^{32} (\theta_i - \mu)(\theta_i - \mu)^T \sim \text{Wishart}_2(32, 32\mathbf{A}^{-1})$$

MCMC Algorithm

$$\theta_i | \hat{\theta}_i, \Sigma, \mathbf{A}, \beta \sim N_2(\mathbf{B}_i \beta + (\mathbf{I} - \mathbf{B}_i) \hat{\theta}_i, (\mathbf{I} - \mathbf{B}_i) \mathbf{V}_i)$$

$$\mathbf{V}_i = \Sigma / n_i, \quad \mathbf{B}_i = \mathbf{V}_i (\mathbf{V}_i + \mathbf{A})^{-1}$$

$$\Sigma | \hat{\Sigma} \sim \text{Inverse-Wishart}_2(N, N \hat{\Sigma})$$

$$\mathbf{A} | \hat{\mathbf{A}} \sim \text{Inverse-Wishart}_2(29, 32 \hat{\mathbf{A}})$$

$$\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}; \quad \text{fix } \beta_1 = \beta_2 = \text{overall average } Y_{ij}.$$

Opponent-Adjusted Season Totals

Offense:

Team	Estimate	Average	Rank(Ave)
Oak	270	293	1
St.L	264	279	2
Buf	259	272	3
Ind	256	262	5
...
Det	200	195	25
AZ	199	190	28
Car	196	186	31
Hou	183	163	32

Estimates are the offensive θ_i estimates added to the defensive mean estimate (112.5 yds) giving the mean passing yards for each team against an average defense.

Averages are the teams' season-average passing yards.

Opponent-Adjusted Season Totals

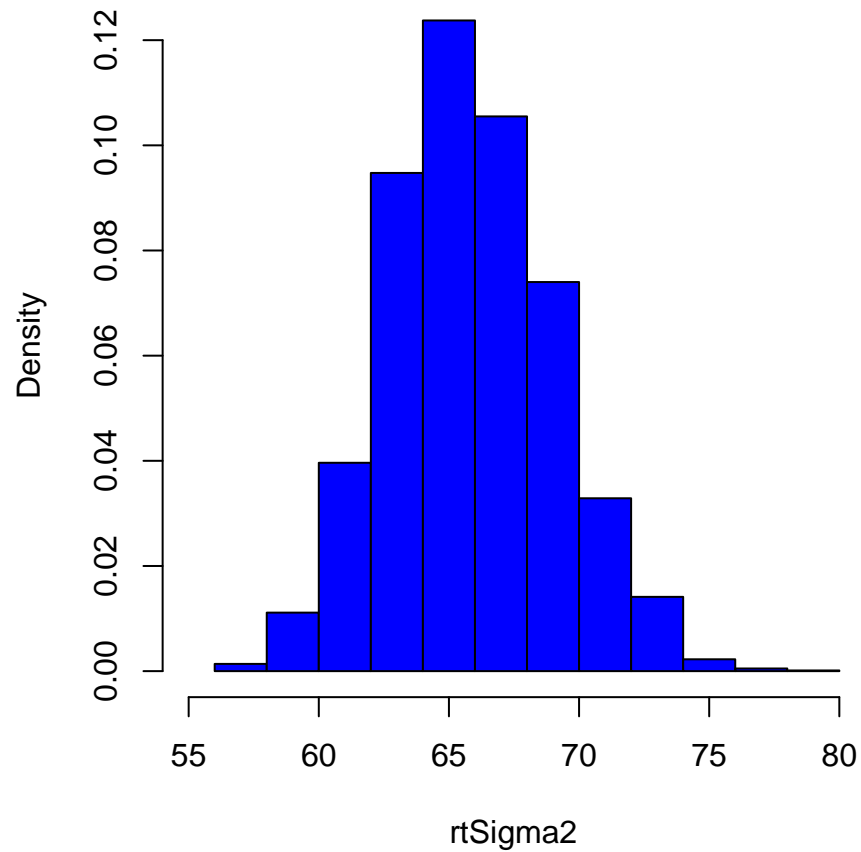
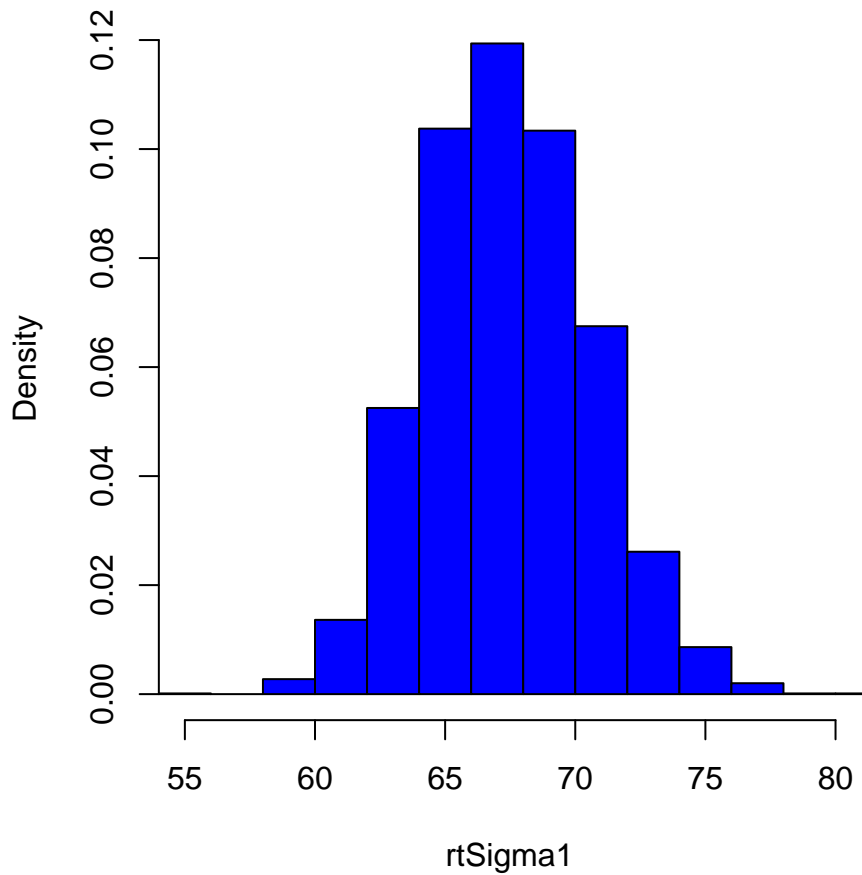
Defense:

Team	Estimate	Average	Rank(Ave)
TB	198	174	1
NYG	212	201	4
NE	212	206	8
Buf	213	205	5
...
KC	247	273	30
MN	249	267	29
SD	253	283	32
Det	257	274	31

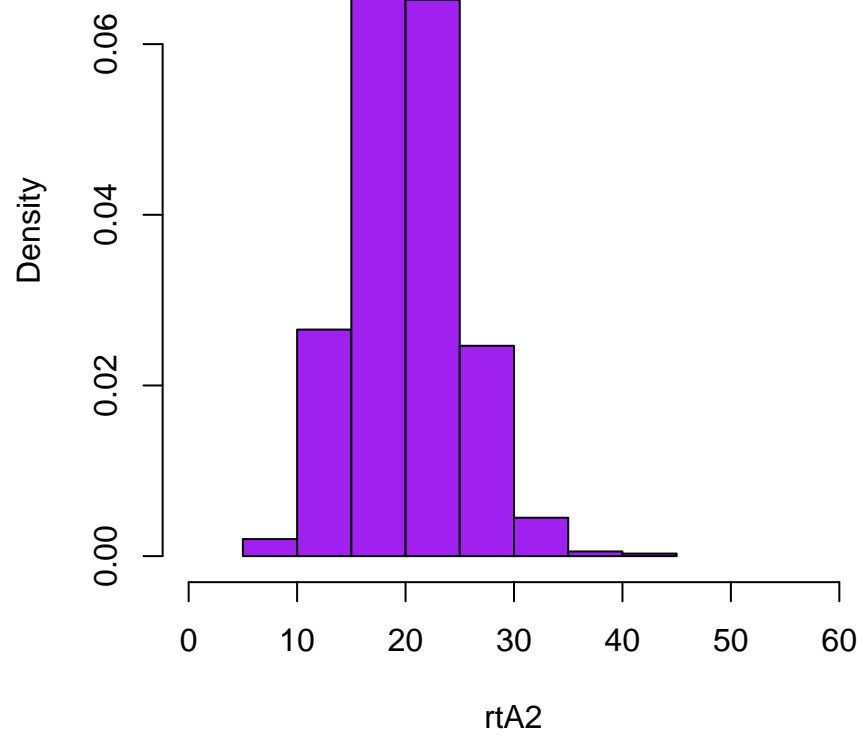
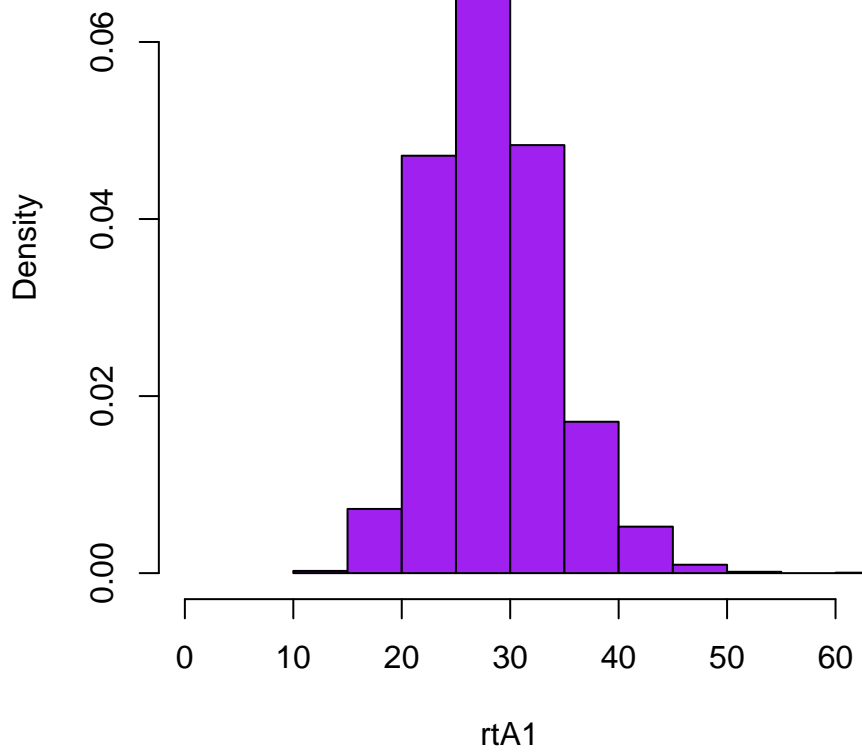
Estimates are the defensive θ_i estimates added to the offensive mean estimate (112.5 yds) giving the mean passing yards allowed for each team against an average offense.

Averages are the teams' season-average passing yards-allowed.

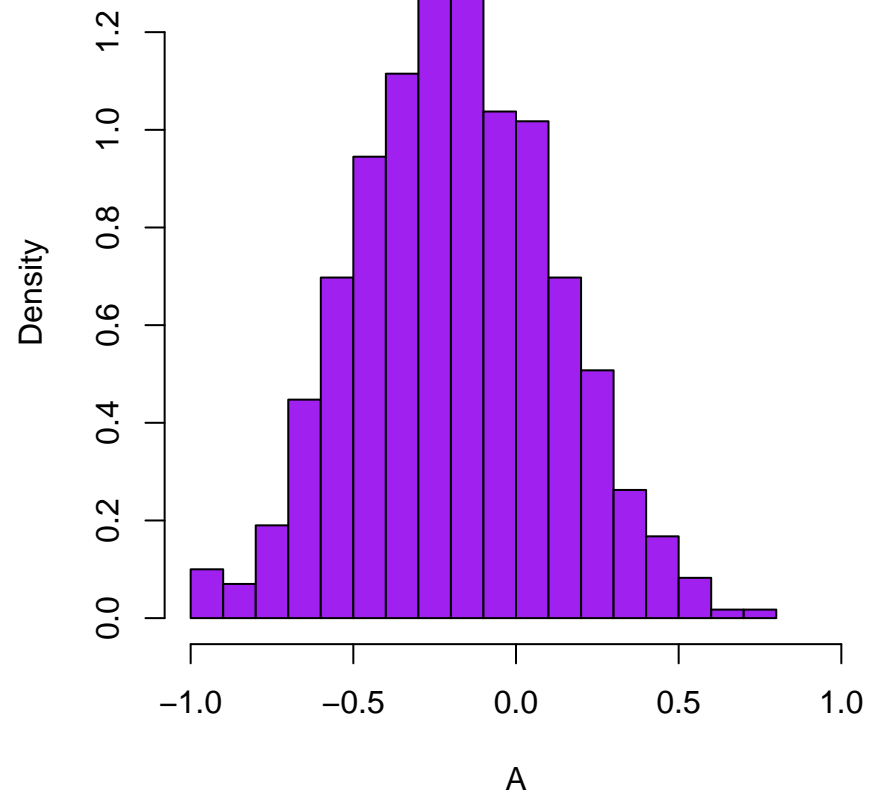
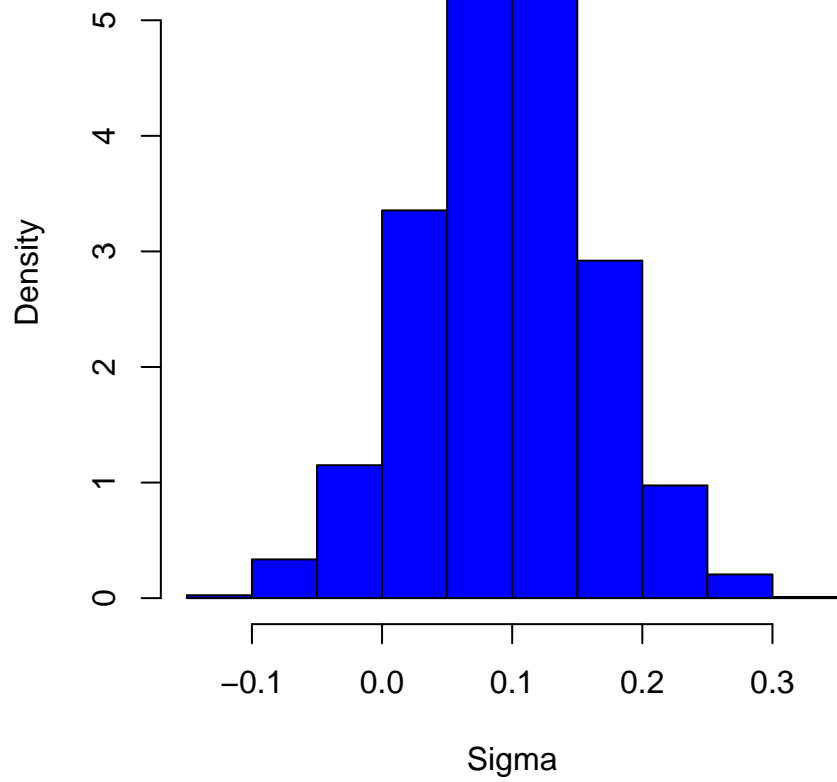
Level-1 Standard Deviations



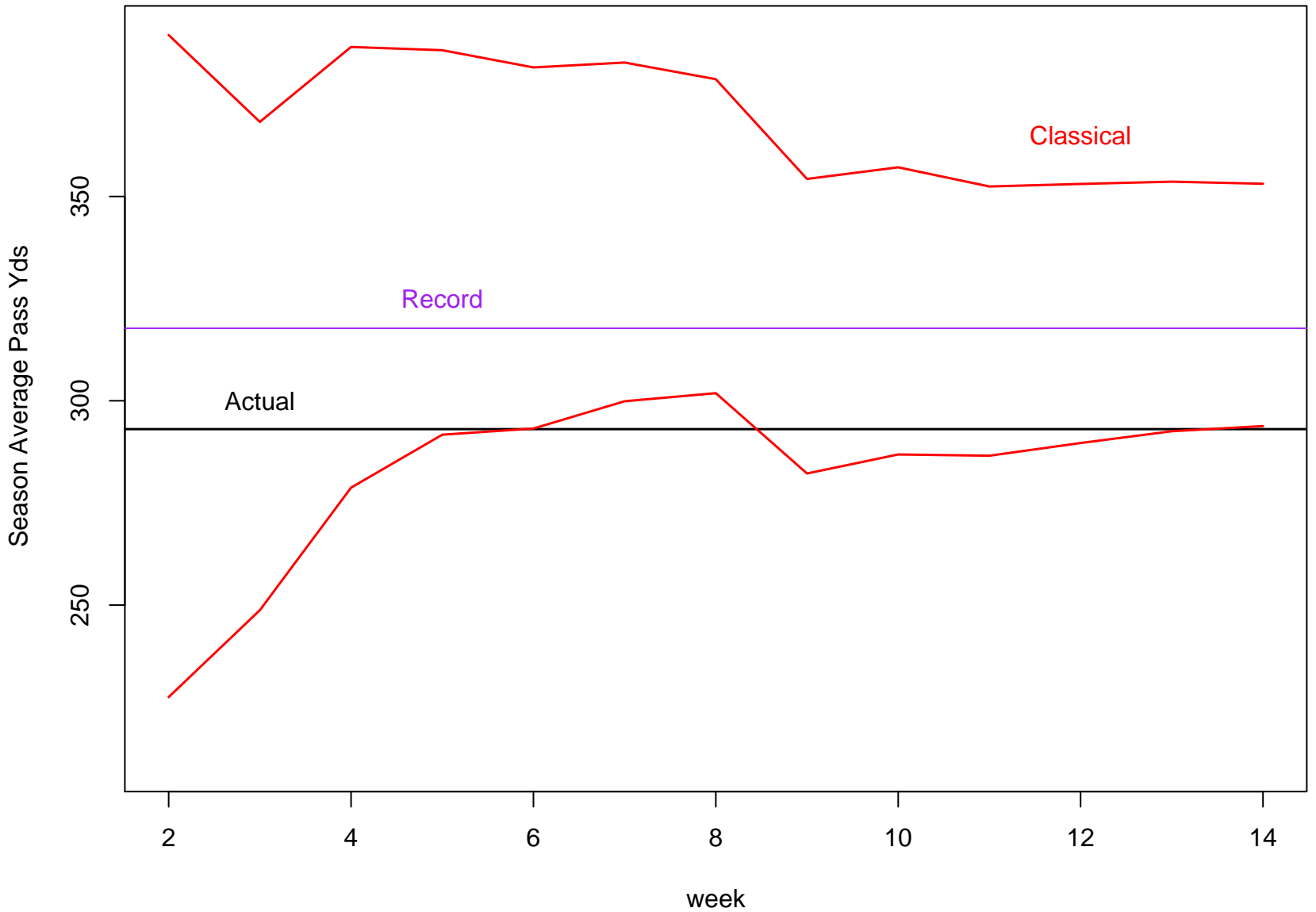
Level-2 Standard Deviations



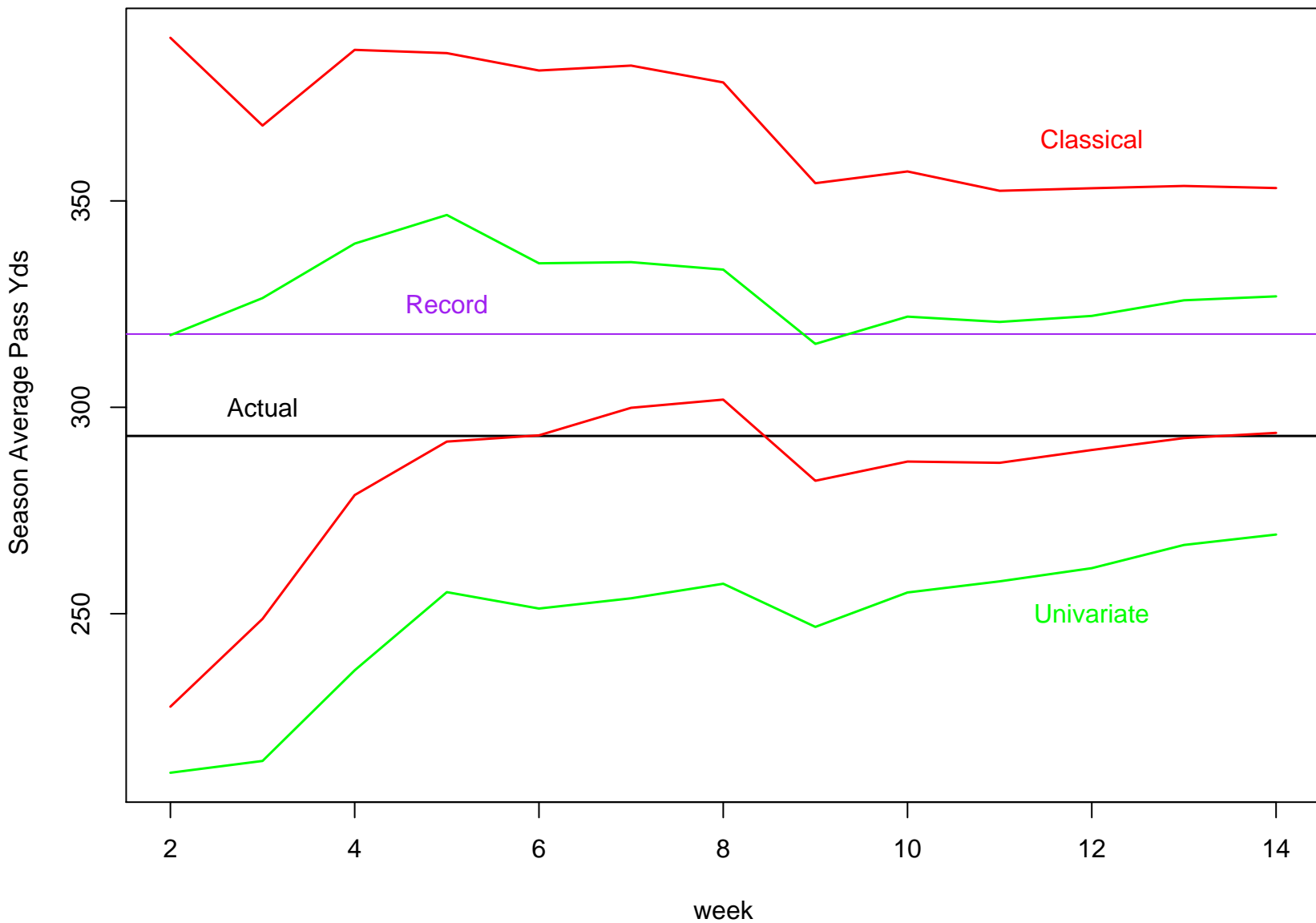
Correlations



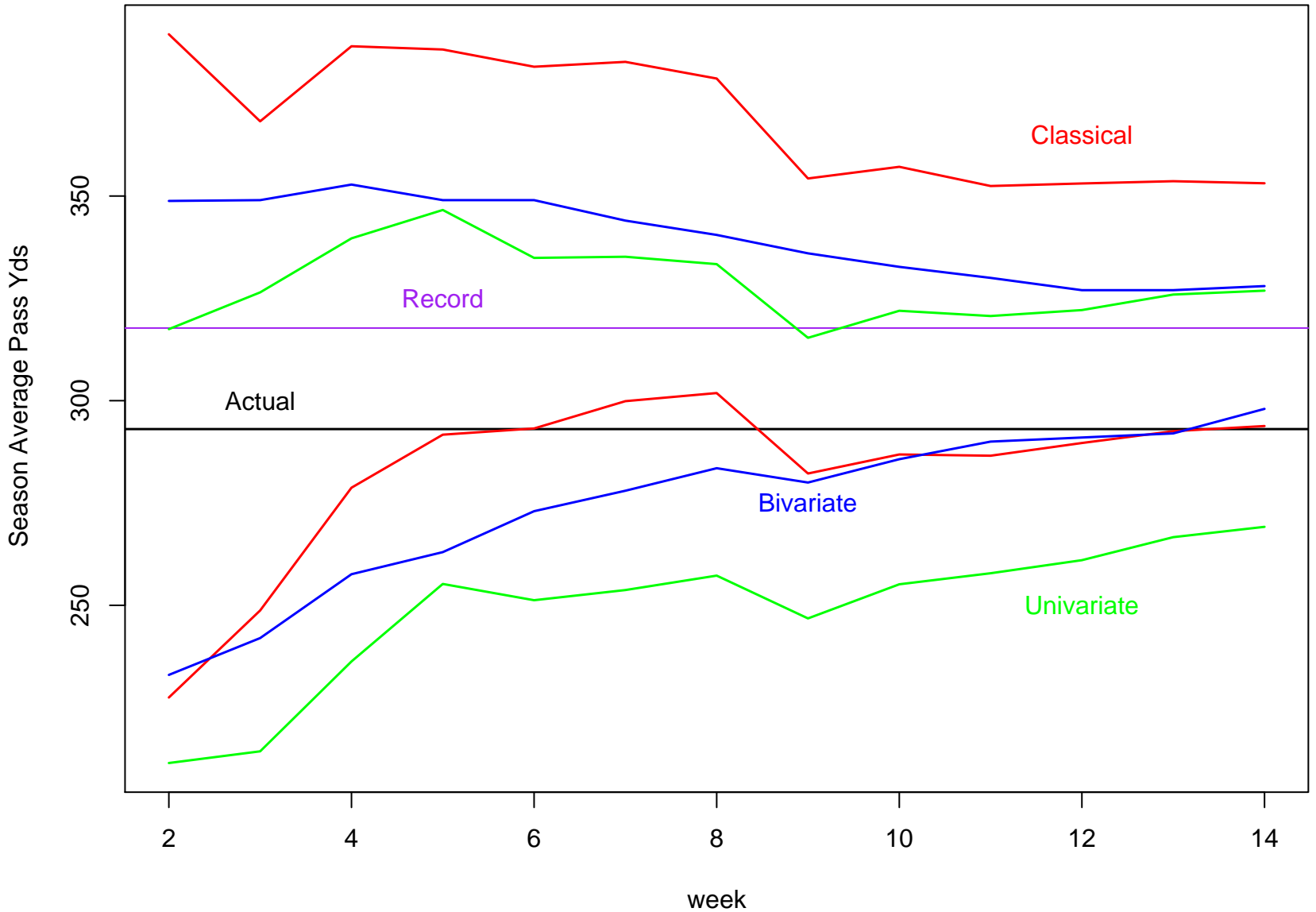
Oakland Projections



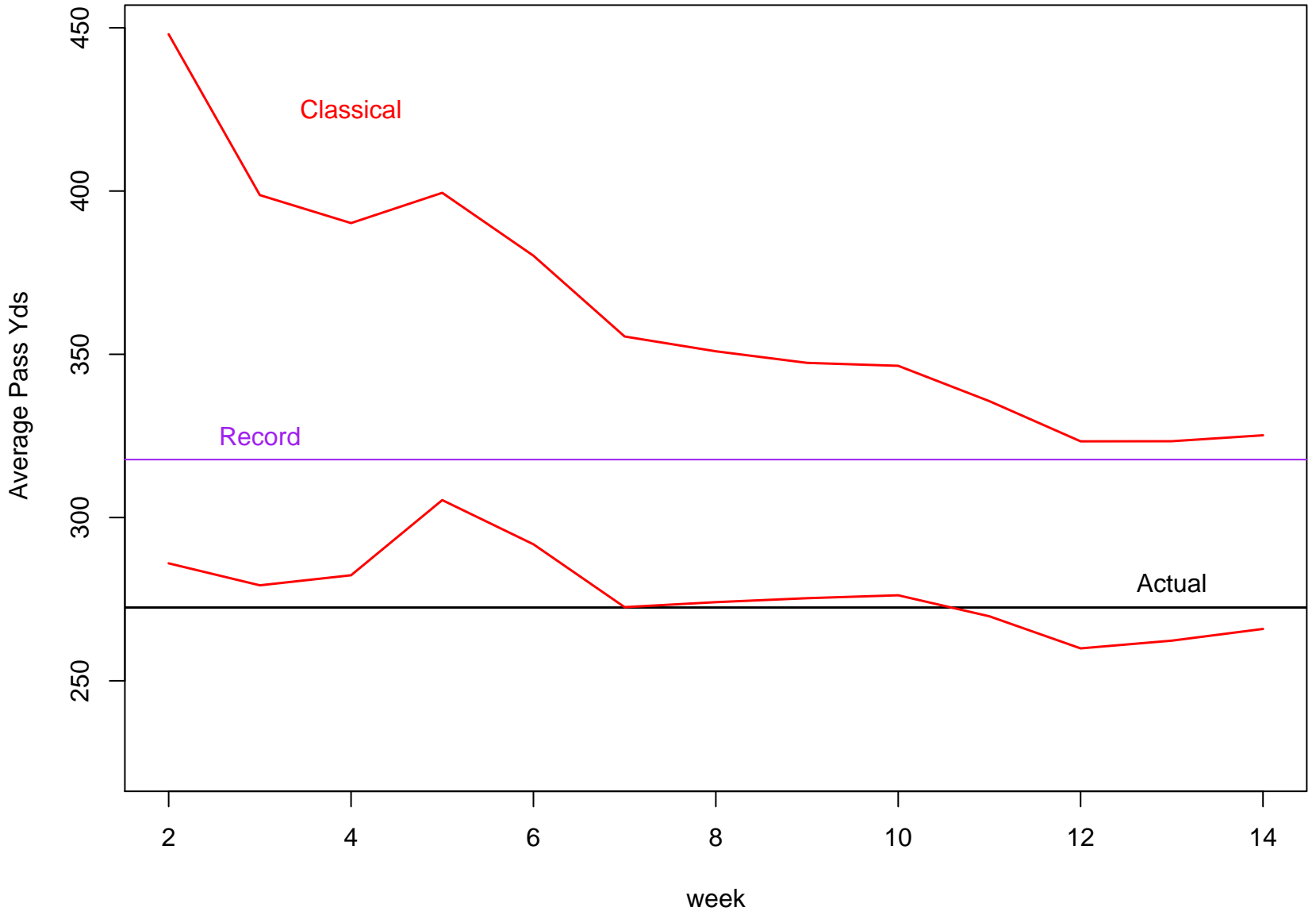
Oakland Projections



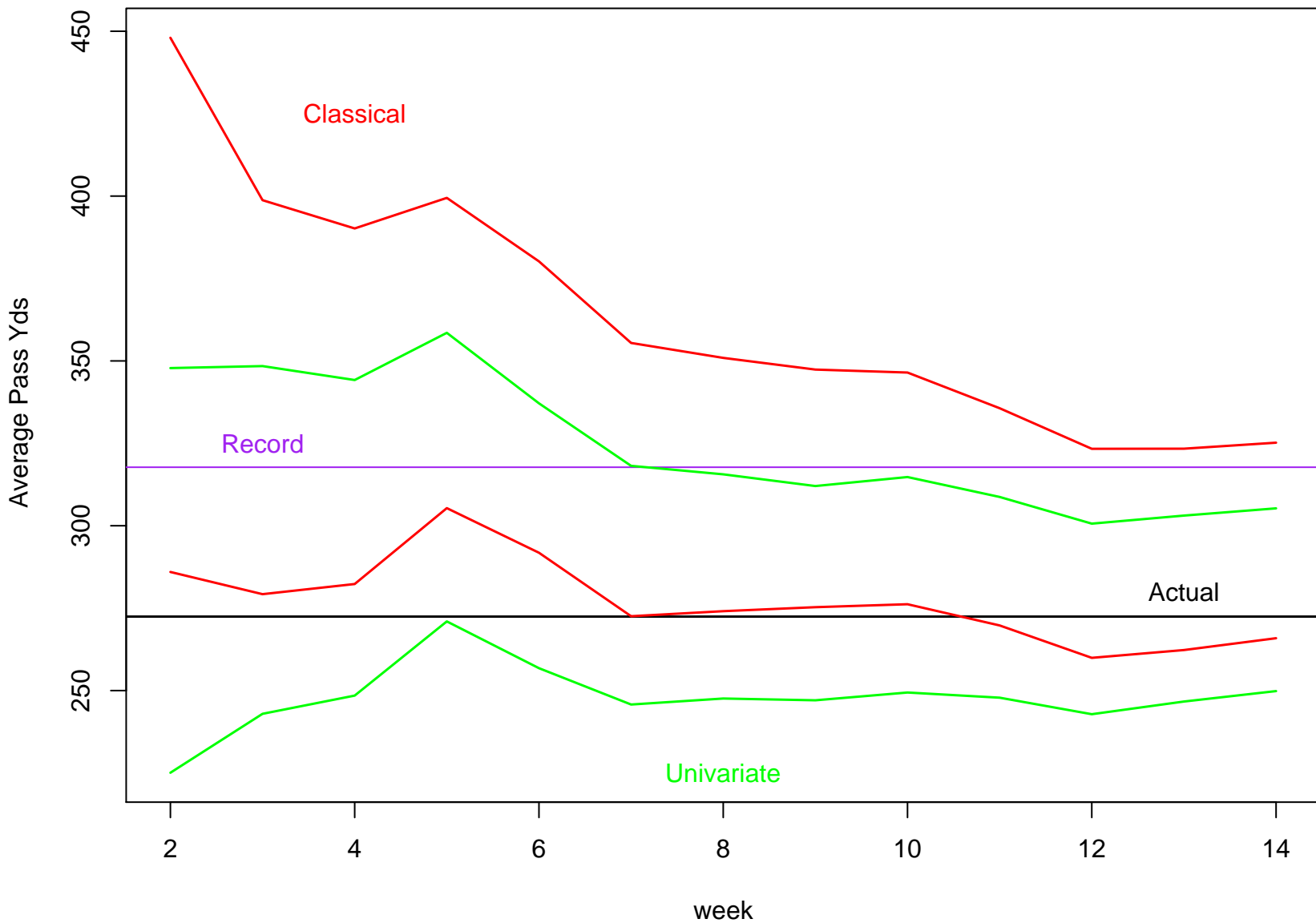
Oakland Projections



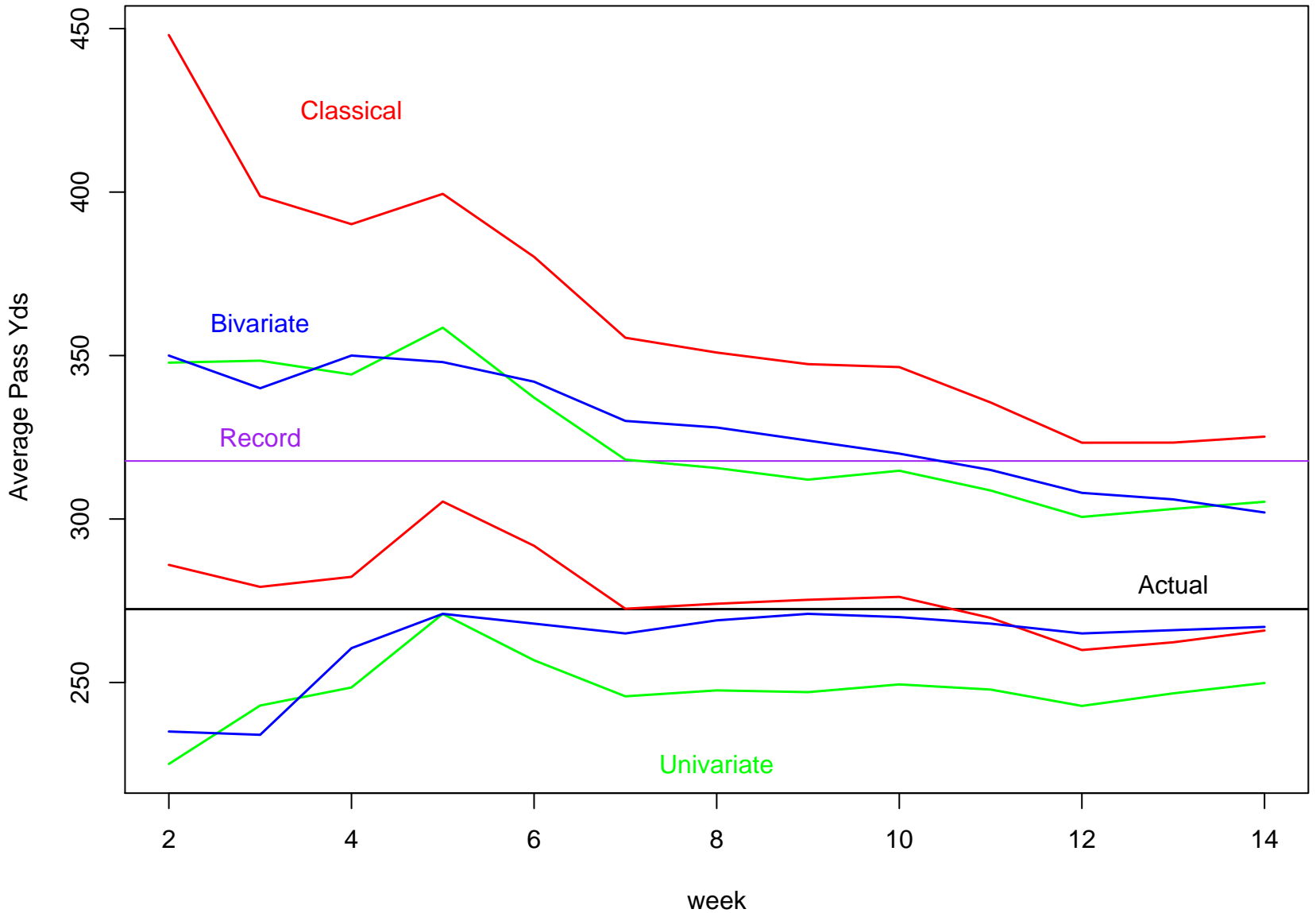
Buffalo Projections



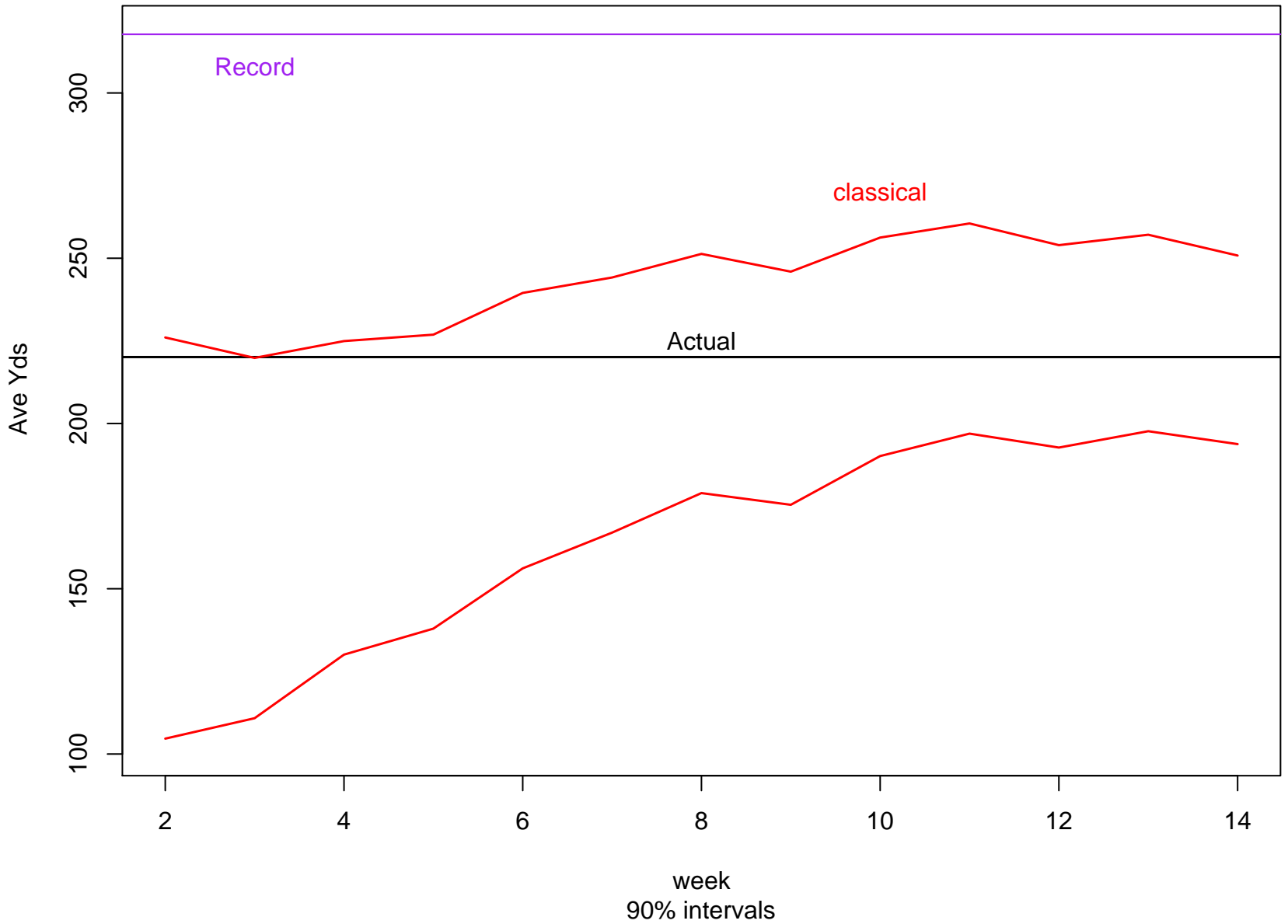
Buffalo Projections



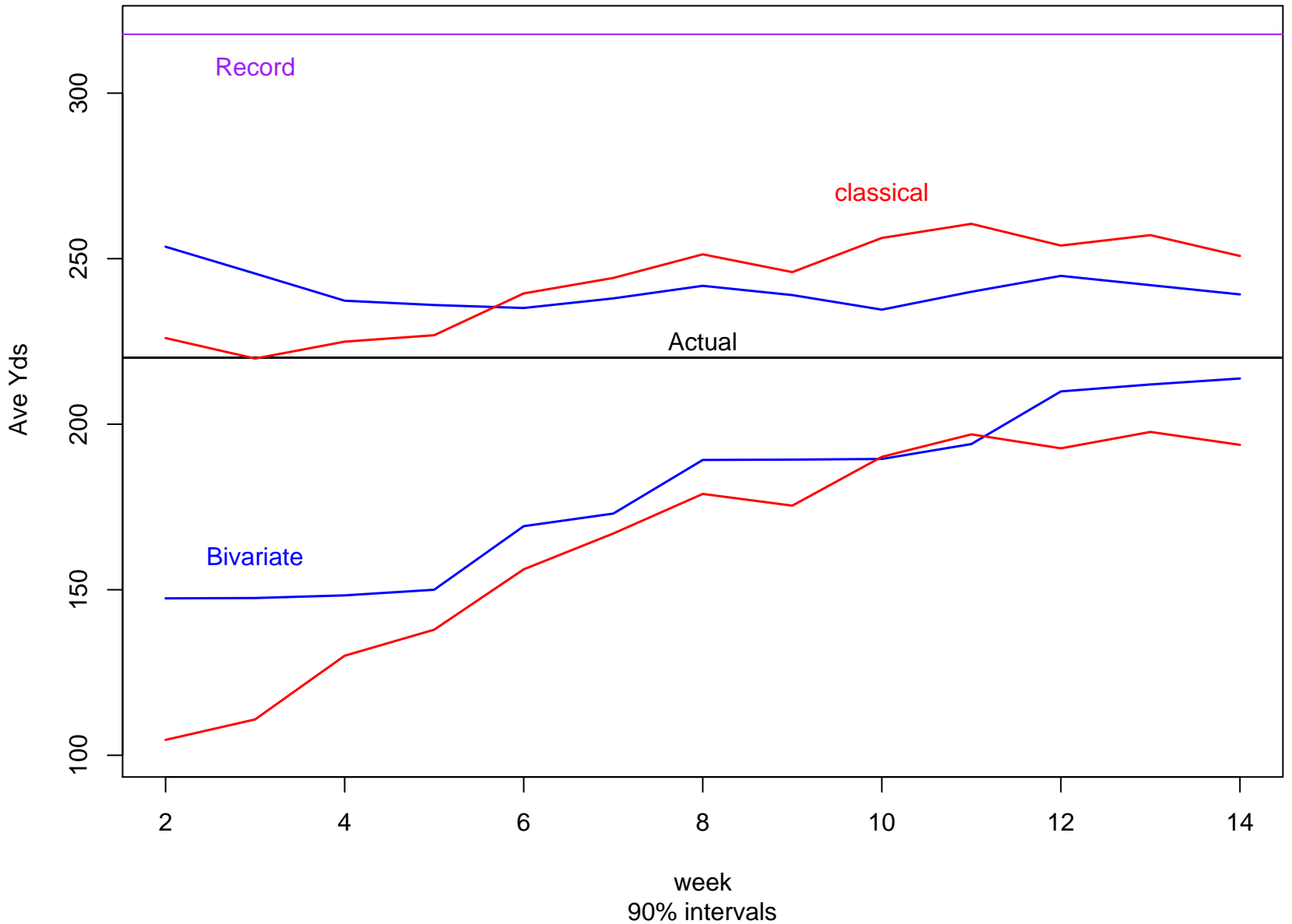
Buffalo Projections



San Francisco Projections



San Francisco Projections



2002 NFL Passing Data

www.swarthmore.edu/NatSci/peverso1/research.html

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