

FUTHER TOPICS: SINGLE VARIABLE CALUCULUS

February 03, 2012

Group Work II

Your Group Number:

Your Names in alphabetical order (**Print last name first**):

1.

2.

3.

4.

Instructions

Write your answer in the space provided after each question.

Show your work to get full credit (unless instructed otherwise) - No credit is given for unsubstantiated answers!

Open Notes - Calculators are allowed!**Question 1**

Evaluate the integral

$$\int \frac{x^3}{\sqrt{x^2 + 16}} dx.$$

Set $x = 4 \tan \theta$, $-\pi/2 < \theta < \pi/2$; $dx = 4 \sec^2 \theta d\theta$

$$\int \frac{x^3}{\sqrt{x^2 + 16}} dx = 64 \int \tan^3 \theta \sec \theta d\theta = 64 \int (\sec^2 \theta - 1) \tan \theta \sec \theta d\theta$$

Set $u = \sec \theta$; $du = \sec \theta \tan \theta d\theta$

$$\int \frac{x^3}{\sqrt{x^2 + 16}} dx = 64 \int (u^2 - 1) du = 64 \left(\frac{\sec^3 \theta}{3} - \sec \theta \right) = 64 \left(\frac{(\frac{\sqrt{x^2+16}}{4})^3}{3} - \frac{\sqrt{x^2 + 16}}{4} \right) + C$$

Question 2

The voltage, V , in an electrical outlet is given as a function of time, t , by the function $V = V_0 \cos(120\pi t)$, where V is in volts and t is in seconds, and V_0 is a positive constant representing the maximum voltage.

- (a) What is the average value of the voltage over 1 second?

$$\text{Average value of } V = \int_0^1 V_0 \cos(120\pi t) dt = V_0 \frac{\sin(120\pi t)}{120\pi} \Big|_0^1 = 0$$

- (b) Engineers do not use the average voltage. They use root mean square voltage defined by

$$\bar{V} = \sqrt{\text{average of } (V^2)}.$$

Find \bar{V} in terms of V_0 . (Take the average over 1 second.)

$$\text{Average of } V^2 = \int_0^1 V_0^2 \cos^2(120\pi t) dt = \frac{V_0^2}{2} \int_0^1 (1 + \cos(240\pi t)) dt = \frac{V_0^2}{2}$$

$$\bar{V} = V_0/\sqrt{2}$$

- (c) The standard voltage in an American house is 110 volts, meaning that $\bar{V} = 110$ volts. What is the amplitude V_0 ?

$$V_0 = 110\sqrt{2} \text{ volts}$$