MATH 15
11/15/10

Name _____

Exam 2

- **Unsupported answers, even if they give the correct final answer, may receive little or no credit!** Be sure to let me know what you're doing and to justify your work!

- Write your answers to the problems in the spaces provided. If you need to continue an answer somewhere else, be sure to tell me where to look for it.

- Calculator use is **not** permitted, but remember: you **DON'T** need to simplify your answers!

- **Good luck!!**

Problem	Points	Score
1	20	
2	12	
3	14	
4	12	
5	18	
6	12	
7	12	
Total	100	

MIDTERM 2 NAME _____

1. (20 points) For each of the following, find $\frac{dy}{dx}$.

a) $y = 2^x \sin x$

b) $y = \arcsin(2x + 1)$

c) $y = \dfrac{3x + 2}{\sqrt{5x^2 - 6}}$

d) $y = \left(\sqrt{\ln x} + \cos(x)\right)^{17}$

e) $x^2y^3 - x = e^{2y}$

2. (12 points)

a) Use the table below to find $(f^{-1})'(1)$. (You may assume that $f(x)$ is actually invertible and differentiable.)

x	$f(x)$	$f'(x)$
-2	4	-2
-1	2	-1.5
0	1	$-.5$
1	-1	-1
2	-3	-2.5

b) Compute $\lim_{x\to 1} \dfrac{\ln x}{\sin(\pi x)}$

c) Compute $\lim_{x\to\infty} \dfrac{e^x}{e^x + x}$

3. Let $f(x) = \sqrt{7+x}$.

a) (9 points) Find the local linearization to $f(x)$ at $x = 2$.

b) (3 points) Use the function from part (a) to estimate $\sqrt{9.05}$

c) (2 points) Why is it better to estimate $\sqrt{9.05}$ using the local linearization to $f(x)$ at 2, rather than at 0?

4. Let $f(x)$ be a function that is defined on the interval $[0, 10]$ and is continuous and differentiable over that entire interval. Suppose we also know:

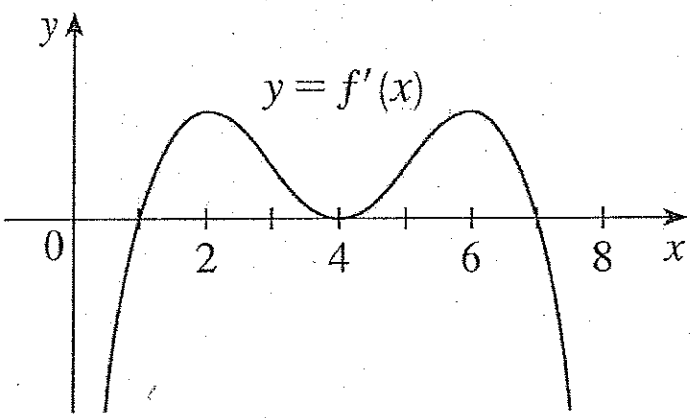
- $f(4) = 0$
- $f'(x) = 0$ when $x = 2$ and $x = 5$
- $f'(x) > 0$ for $0 \leq x < 2$ and for $2 < x < 5$
- $f'(x) < 0$ for $5 < x \leq 10$
- $f''(x) = 0$ when $x = 2$ and $x = 4$
- $f''(x) < 0$ for $0 \leq x < 2$ and $4 < x \leq 10$
- $f''(x) > 0$ for $2 < x < 4$

a) (2 points) Where are the critical points of $f(x)$?

b) (8 points) Find the local maxima and minima of $f(x)$. (Be sure to say which are which, and **don't forget** that endpoints can be local extrema too!)

c) (2 points) Where are the inflection points of $f(x)$?

5. (24 points) The graph below is of $f'(x)$, the **DERIVATIVE** of $f(x)$.



a) (2 points) What are the critical points of $f(x)$?

b) (2 points) At what value(s) of x does $f(x)$ have a local maximum?

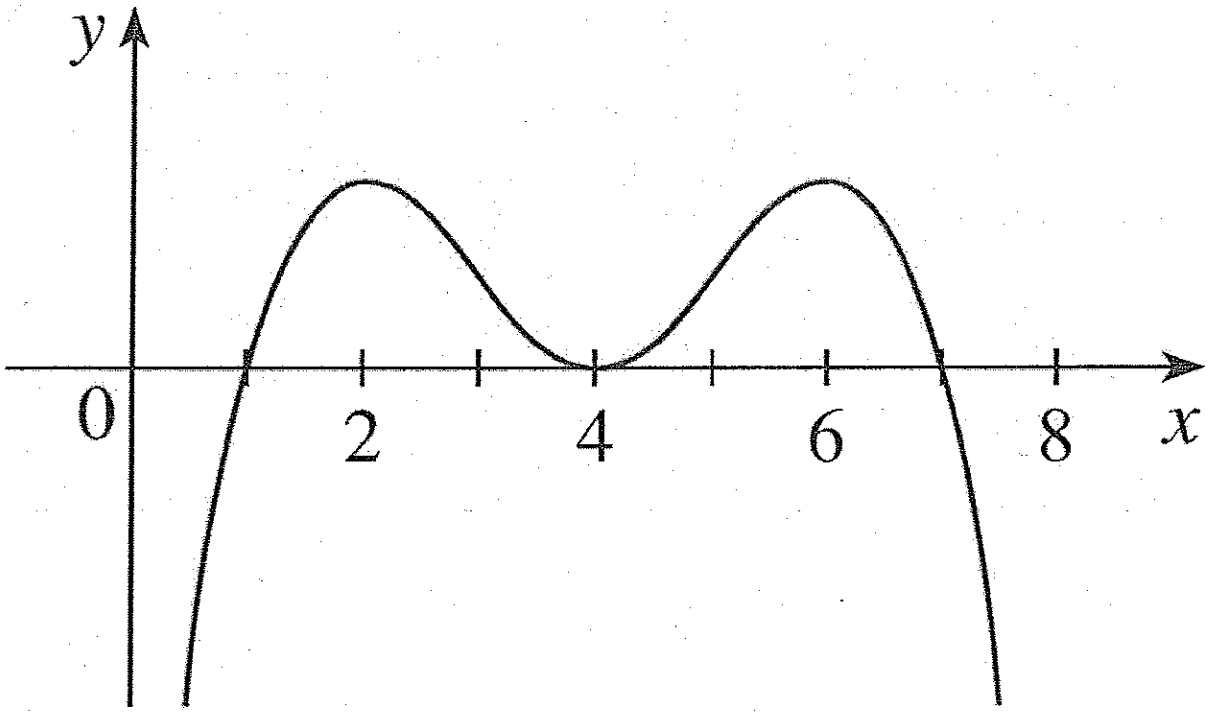
c) (2 points) At what value(s) of x does $f(x)$ have a local minimum?

d) (2 points) On what interval(s) is $f(x)$ increasing?

e) (2 points) On what interval(s) is $f(x)$ concave down?

f) (2 points) Find the inflection point(s) of $f(x)$.

Here is that graph of $f'(x)$ again:



g) (6 points) Add a sketch of the graph of $f(x)$ to the above picture.

6. (12 points) You have just invented a new chocolate hummus dip, and you open a stand in front of Tarble to sell this stuff by the jar. Somehow a rumor gets started (certainly not traceable to you) that your concoction boosts exam scores, and sales take off. You are able to produce only 500 jars a day, and if you charge \$1.00 for each jar, all 500 will sell. For each nickel that you increase the price, you sell two fewer jars. Assuming that you have a fixed cost per day of \$200, and the cost per jar is 50 cents, determine the price for which you should sell your dip in order to maximize your profits. (Assume that you produce exactly the number of jars that you sell–after all, there isn't much storage space in your dorm room.) Be sure to justify why this price *maximizes* your profits!

7. (12 points) Suppose you want a cylindrical plastic container to hold 240 cubic centimeters (approximately 8 ounces) of chocolate hummus. What dimensions of the container will minimize the amount of plastic used to make the container?