Exam 1
October 5, 2010

- Unsupported answers, even if they give the correct final answer, may receive little or no credit! Be sure to let us know what you're doing and to justify your work!
- Write your answers to the problems in the spaces provided. If you need to continue an answer somewhere else, be sure to tell us where to look for it.
- Calculator use is not permitted, but remember: you DON'T need to simplify your answers!
- Good luck!!

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
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<tbody>
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<td>1</td>
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<td>Total</td>
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</table>
1. (14 points) Compute the following limits, or show that they don't exist.

a) \( \lim_{x \to 2} \frac{2x^2 + x - 1}{x^2 - 1} \)

b) \( \lim_{x \to 1^+} \frac{2x^2 + x - 1}{x^2 - 1} \)

c) \( \lim_{x \to 1} \frac{2x^2 + x - 1}{x^2 - 1} \)

d) \( \lim_{x \to 1} \frac{2x^2 + x - 1}{(x - 1)^2} \)
e) \( \lim_{x \to -1} \frac{2x^2 + x - 1}{x^2 - 1} \)

f) \( \lim_{x \to \infty} \frac{2x^2 + x - 1}{x^2 - 1} \)

g) \( \lim_{x \to \infty} \frac{2x^2 + x - 1}{x - 1} \)
2. (10 points) Using the definition of the derivative, find the derivative of \( f(x) = \frac{1}{1 - x} \).
3. (15 points) A giddily gleeful student, after acing her Math 15 exam, hurls a somewhat large calculus book directly upward from the ground. Its position at time $t$ is given by $s(t) = 96t - 16t^2$, where $t$ is the number of seconds, and $s(t)$ is the number of feet above the ground. Compute the following (no simplification necessary!), giving units:

a) the velocity of the book after 1.5 seconds

b) the acceleration of the book at $t = 1.5$ seconds

c) the maximum height the book reaches (hint: what is the velocity at the maximum height?)

d) the average velocity of the book between $t = 1$ and $t = 2$

e) the instantaneous rate of change of the acceleration at $t = 4$. 
4. Pretend that I start the following (sadly, false) rumor around campus: any student enrolled in Math 15 does not have to pay tuition that semester. Let \( R(t) \) denote the number of Swarthmore students who have heard this rumor after \( t \) days. Suppose \( R(7) = 512 \) and \( R'(7) = 107 \).

a) (1 point) What are the units of \( R(7) \)?

b) (2 points) What are the units of \( R'(7) \)?

c) (6 points) Explain the practical significance of \( R(7) = 312 \) and \( R'(7) = 107 \).

d) (4 points) Suppose \( R''(14) \) is negative. Explain what this says about the spread of the rumor.
5. (6 points) Consider the following table of values:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>.2</th>
<th>.3</th>
<th>.4</th>
<th>.5</th>
<th>.6</th>
<th>.7</th>
<th>.8</th>
<th>.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>2.8</td>
<td>3.0</td>
<td>3.2</td>
<td>3.2</td>
<td>2.8</td>
<td>2.5</td>
<td>2.3</td>
<td>2.2</td>
<td>2.1</td>
<td>2.4</td>
<td>3.0</td>
</tr>
</tbody>
</table>

a) What does the sign of $f'(0.5)$ seem to be? Why?

b) What does the sign of $f''(0.5)$ seem to be? Why?

6. (12 points) Suppose that $g(x)$ is differentiable on the interval $[0, 3]$, $g(1) = 4$ and $g(2) = 17$. Circle all the statements below that MUST be true.

- $g(x)$ is continuous at $x = 2$
- $\lim_{h \to 0} \frac{g(1 + h) - g(1)}{h}$ exists
- $\lim_{h \to 0} \frac{g(1 + h) - g(1)}{h} = 13$
- $\lim_{x \to 1} g(x) = 4$
- $g'(x) \geq 0$ on $(1, 2)$
- $g(x) = 15$ somewhere on the interval $[1, 2]$
7. Below is the graph of a function \( y = f(x) \).

\[ f(x) \]

\[ \begin{array}{c|c|c|c|c|c|c|c|c|}
\hline
& -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
-1 & & & & & & & & \\
0 & & & & & & & & \\
1 & & & & & & & & \\
2 & & & & & & & & \\
3 & & & & & & & & \\
4 & & & & & & & & \\
5 & & & & & & & & \\
\hline
\end{array} \]

a) (2 points) Where on the interval \([0,6]\) does \( f(x) \) appear to be continuous?

b) (2 points) Where on the interval \([0,6]\) does \( f(x) \) appear to be differentiable?

c) (2 points) Where on the interval \([0,6]\) does \( f(x) \) appear to be defined?

d) (2 points) Does \( \lim_{x \to 3} f(x) \) exist? If so, estimate; if not, say why not.

e) (4 points) Estimate \( f'(4) \). Be sure to explain how you are getting your estimate!
8. (8 points) The following graph shows the DERIVATIVE of \( f(x) \).

\[
y = f'(x)
\]

a) Where is \( f(x) \) increasing?

b) Where is \( f(x) \) concave up?
9. (10 points) Sketch a function \( f(x) \) that satisfies the following specifications. Assume \( f(x) \) is differentiable unless otherwise noted. Be sure your graph clearly exhibits each of the listed features.

- \( f \) is not differentiable at \( x = 0 \) and \( x = 1 \).
- \( \lim_{x \to 0^-} f(x) = -\infty \)
- \( \lim_{x \to 0^+} f(x) = +\infty \)
- \( f'(x) = 1 \) for \( x > 1 \).
- \( f'(x) = 0 \) at \( x = -1 \) and \( x = -3 \).
- \( f''(x) < 0 \) for \( x < -4 \) and \( -2 < x < 0 \).
- \( \lim_{x \to -\infty} f(x) = 0 \)