MATH 15
Grood/Johnson

Name _____

Exam 1

October 5, 2010

- **Unsupported answers, even if they give the correct final answer, may receive little or no credit!** Be sure to let us know what you're doing and to justify your work!
- Write your answers to the problems in the spaces provided. If you need to continue an answer somewhere else, be sure to tell us where to look for it.
- Calculator use is **not** permitted, but remember: you **DON'T** need to simplify your answers!
- **Good luck!!**

Problem	Points	Score
1	14	
2	10	
3	15	
4	13	
5	6	
6	12	
7	12	
8	8	
9	10	
Total	100	

1. (14 points) Compute the following limits, or show that they don't exist.

a) $\lim_{x \to 2} \dfrac{2x^2 + x - 1}{x^2 - 1}$

b) $\lim_{x \to 1^+} \dfrac{2x^2 + x - 1}{x^2 - 1}$

c) $\lim_{x \to 1} \dfrac{2x^2 + x - 1}{x^2 - 1}$

d) $\lim_{x \to 1} \dfrac{2x^2 + x - 1}{(x - 1)^2}$

e) $\lim_{x \to -1} \dfrac{2x^2 + x - 1}{x^2 - 1}$

f) $\lim_{x \to \infty} \dfrac{2x^2 + x - 1}{x^2 - 1}$

g) $\lim_{x \to \infty} \dfrac{2x^2 + x - 1}{x - 1}$

2. (10 points) **Using the definition of the derivative**, find the derivative of $f(x) = \dfrac{1}{1-x}$.

3. (15 points) A giddily gleeful student, after acing her Math 15 exam, hurls a somewhat large calculus book directly upward from the ground. Its position at time t is given by $s(t) = 96t - 16t^2$, where t is the number of seconds, and $s(t)$ is the number of feet above the ground. Compute the following (**no** simplification necessary!), **giving units**:

a) the velocity of the book after 1.5 seconds

b) the acceleration of the book at $t = 1.5$ seconds

c) the maximum height the book reaches (hint: what is the velocity at the maximum height?)

d) the average velocity of the book between $t = 1$ and $t = 2$

e) the **instantaneous rate of change** of the acceleration at $t = 4$.

4. Pretend that I start the following (sadly, false) rumor around campus: any student enrolled in Math 15 does not have to pay tuition that semester. Let $R(t)$ denote the number of Swarthmore students who have heard this rumor after t days. Suppose $R(7) = 512$ and $R'(7) = 107$.

a) (1 point) What are the units of $R(7)$?

b) (2 points) What are the units of $R'(7)$?

c) (6 points) Explain the practical significance of $R(7) = 312$ and $R'(7) = 107$.

d) (4 points) Suppose $R''(14)$ is negative. Explain what this says about the spread of the rumor.

5. (6 points) Consider the following table of values:

x	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
$f(x)$	2.8	3.0	3.2	3.2	2.8	2.5	2.3	2.2	2.1	2.4	3.0

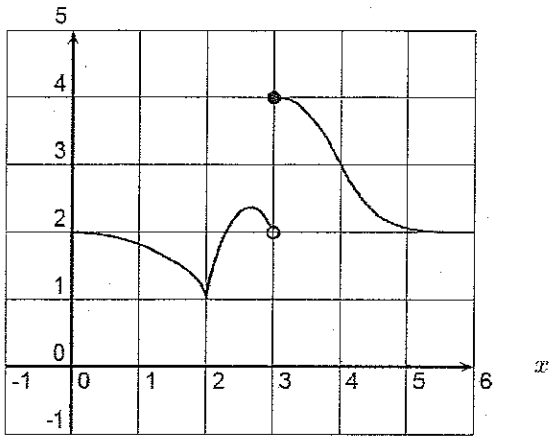
a) What does the sign of $f'(.5)$ seem to be? Why?

b) What does the sign of $f''(.5)$ seem to be? Why?

6. (12 points) Suppose that $g(x)$ is differentiable on the interval $[0, 3]$, $g(1) = 4$ and $g(2) = 17$. Circle *all* the statements below that **MUST** be true.

- $g(x)$ is continuous at $x = 2$
- $\lim_{h \to 0} \dfrac{g(1+h) - g(1)}{h}$ exists
- $\lim_{h \to 0} \dfrac{g(1+h) - g(1)}{h} = 13$
- $\lim_{x \to 1} g(x) = 4$
- $g'(x) \geq 0$ on $(1, 2)$
- $g(x) = 15$ somewhere on the interval $[1, 2]$

7. Below is the graph of a function $y = f(x)$.



a) (2 points) Where on the interval $[0, 6]$ does $f(x)$ appear to be continuous?

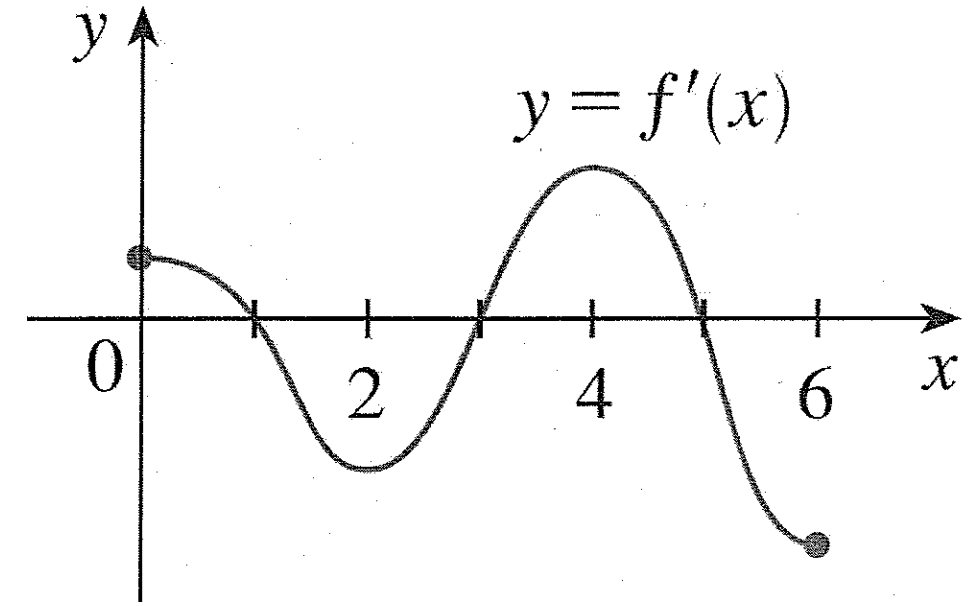
b) (2 points) Where on the interval $[0, 6]$ does $f(x)$ appear to be differentiable?

c) (2 points) Where on the interval $[0, 6]$ does $f(x)$ appear to be defined?

d) (2 points) Does $\lim_{x \to 3} f(x)$ exist? If so, estimate; if not, say why not.

e) (4 points) Estimate $f'(4)$. Be sure to explain how you are getting your estimate!

8. (8 points) The following graph shows the **DERIVATIVE** of $f(x)$.



a) Where is $f(x)$ increasing?

b) Where is $f(x)$ concave up?

9. (10 points) Sketch a function $f(x)$ that satisfies the following specifications. Assume $f(x)$ is differentiable unless otherwise noted. Be sure your graph clearly exhibits each of the listed features.

- f is not differentiable at $x = 0$ and $x = 1$.
- $\lim_{x \to 0^-} f(x) = -\infty$
- $\lim_{x \to 0^+} f(x) = +\infty$
- $f'(x) = 1$ for $x > 1$.
- $f'(x) = 0$ at $x = -1$ and $x = -3$.
- $f''(x) < 0$ for $x < -4$ and $-2 < x < 0$.
- $\lim_{x \to -\infty} f(x) = 0$