SI Quantities

Base Units
- length (m)
- time (s)
- mass (kg)
- current (A)

Derived Units
- force \( \frac{N}{a} = \frac{kg m}{s^2} \)
- energy \( J = Nm = kg m^2 \)
- power \( W = \frac{j}{s} \)
- torque \( (Nm) \)

Today:
- Motors
- 3 take-away points

Most robots in the world move w/ the use of

3 Takeaways from Brushed DC Motors

1. Motor = resistor & voltage drop
2. Linear relationship between torque & speed
3. Quadratic relationship between torque & power
   Also: gears help?

Electrically

\[ V = IR + e \] (due to KVL)

\[ e = \omega Ke \]

\[ R \] is the motor's internal resistance (mostly from coils)

\( e \) is the back-EMF (electromotive force) potential

\( \omega \) is speed-dependent, constant

\( k \) is back-emf constant
\[
T = K_e I
\]

\[
I = \frac{V}{K} \quad \Rightarrow \quad V = \frac{K}{R} I + K w = -\frac{R}{K^2} T + \frac{V}{K} \quad \Rightarrow \quad \text{Linear relationship between torque and speed}
\]

\[W_{\text{max}} = \frac{V}{K} \quad @ \quad T = 0\]

\[T_s = \text{static torque where} \quad W \to 0\]

\[T_s = K I = \frac{K V}{R}\]

Now look at power:

\[P_e = V I = \frac{V^2}{R} \quad \text{(electrical)}\]

\[P_m = \frac{T w}{(w)} \quad (N_m) \quad (\text{rad/s}) \quad \text{(mechanical)}\]

\[P_e = P_m? \quad \text{NO...}\]

In fact...

\[P_m = \eta P_e \quad \eta < 1\]

\[P_m = \frac{T w}{T_{\text{max}}} = T \left( -\frac{R}{K^2} T + \frac{V}{K} \right) \quad \Rightarrow \quad \text{power is quadratic in torque}\]

at \(T = 0\), \(P_m = 0\)

at \(T = T_s\), \(P_m = 0\)

\(P_{\text{max}}\) is achieved at \(T_s/2\)

Why are motors bad for robots?

peak efficiency at low torque / high speed
robust efficiency at high torque / low speed

Fix with gears:

\[\omega_2 = \omega_1 \frac{r_1}{r_2}\]

\[G = \frac{r_1}{r_2} \quad \Rightarrow \quad \text{multiplies speed, divides torque}\]

output: input > 1

Digital Control of DC Motors

2 problems:

1. change supply voltage \(v\)
   2. change motor direction

1. use D/A converter - rarely used - requires too many wires
   2. use PWM

\[\text{PWM} = \text{pulse width modulation}\]

60% of 12V duty cycle
**Random Thoughts**
- H-bridges
- configuration spaces
- kinematic systems

**H-Bridge**

**Random Thoughts**
- projects (due Tues 9/27)
- time management
- terminal elective (where you go afterwards is up) to you?

**Configuration Spaces**

A configuration $q$ contains the parameters that define the state of the robot.

- For a turtlebot $q = (x, y, \theta)$

The configuration space is the set of all configurations $X = \{ q \mid q \text{ is a robot state} \}$

E.g. a 2 degrees of freedom planar arm $q = (\theta_1, \theta_2)$

A configuration space is not necessarily just $R^n$ because topology matters.

**Random Thoughts**
- $\Theta_1 \approx \frac{\pi}{6}$
- $\Theta_2 \approx \frac{\pi}{2}$
- $\Theta_3 = \frac{11\pi}{6}$
- $\Theta_4 = \pi/2$

**Common Topological Spaces for Robotics**

$R^n \Rightarrow$ cartesian coords. in n dimensions

$S^1 \Rightarrow$ unit circle ($\Theta$ = rad)

$SO(3) \Rightarrow$ rotations in 3D

- Example: $R^2 \times S^1 \Rightarrow$ rigid transform on 2D

 Kinematic Systems

- A Kinematic system is a mapping $\dot{q} = \sum_{i=1}^{m} V_i(q)w_i = \left[ V_1(q) \ V_2(q) \ \cdots \ V_m(q) \right] \left[ w_1 \ w_2 \ \cdots \ w_m \right]^T$

- Each $w$ is a control that acts as a coeff on a basis $\{e_i(q)\}$

- Each $V_i : X \rightarrow X$ is a function that maps a configuration to a velocity in configuration space - vector field.
So motion is linear in basis functions & controls

- Basis functions $V_i(q)$ are not themselves necessarily linear

$$\begin{align*}
\dot{x}_R &= \frac{r}{a} (V_L + V_R) \\
\dot{\theta} &= \frac{r}{ad} (V_R - V_L)
\end{align*}$$

robot frame forward $\rightarrow$ world frame velocity

ex: Turtlebot

$$\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} = 
\begin{bmatrix}
\frac{r/2 \cos \theta}{r/2 \sin \theta} & \frac{-r/2 \sin \theta}{r/2 \cos \theta} \\
\frac{-r/2 \sin \theta}{r/2 \cos \theta} & \frac{r/2 \cos \theta}{r/2 \sin \theta} \\
\frac{r/2 \cos \theta}{r/2 \sin \theta} & \frac{-r/2 \sin \theta}{r/2 \cos \theta}
\end{bmatrix}
\begin{bmatrix}
V_L \\
V_R
\end{bmatrix}
$$

$3 \times 1$ $3 \times 2$ $2 \times 1$

$\sim f$ is linear if $f(ax + by) = af(x) + bf(y)$

**Kinematic Constraints**

- Define a kinematic constraint as an equality relationship that governs the motion thru $q$-space

$$f(q, \dot{q}) = 0$$

$$\dot{p} \cdot b_2 = 0$$

$$-x \sin \theta + y \cos \theta = 0$$

$$f(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta}) = -x \sin \theta + y \cos \theta = 0$$