E28: Mobile Robotics

- Class Website
- TBA - Class at end of semester - Guest Lecture?
- No textbook - Reading online
- Piazza - Post questions to classmates/professor
  - Participation points
- NO EXAMS! Projects instead.

Grading:
- Weekly homeworks, 4 semester projects,
  - 1 final project, participation
- Collaboration on assignments - allowed, but give credit to people you talk to
- Cite sources
- Don't cheat...

Projects - will be with a group
- No solutions on Piazza

- Late Policy
- Students get 2 free late homeworks without policy
- Homework assignments may be turned in up to 4 days late for half credit

--- Pre Test ---

- Robot Movies -
  - Little dog - learning how to walk (DARPA)
    - Motion planning
    - Maze video
  - Shaky - 60s - first AI robot
  - Stanford Driverless car race (DARPA)

- We will use TurtleBot 2
Orthogonal Transformations

- spatial relationships in 3D

**Def:** An $n \times n$ matrix $A$ is an orthogonal transformation if and only if:

- It has $n$ mutually perpendicular rows or columns with unit length.
- $T$ rows must be independent (can't be multiples of each other).

**Ex:** $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ → linearly dependent

$\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$ → independent but not $T$

- to be perpendicular, the dot product must be $0$

**dot product:** $x \cdot y = \sum_{i=1}^{n} x_i y_i$

$x \cdot y = 0 \iff x \perp y$ (perp.)

- rows/columns must have unit length

$\Rightarrow \|x\|^2 = \sum_{i} x_i^2 = x \cdot x$

- The rows or columns of $A$ form an orthonormal basis of $\mathbb{R}^n$

- basic for space - set of vectors that can combine to create any vector in a space

- basically first point with more words

* More about transpose on next page

- $A A^T = A^T A = I$ (transpose swaps rows and columns)

- $A^{-1} = A^T$

**Ex:** $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 5 & 6 & 6 \end{bmatrix}$
more about $AA^T = A^T A = I$ & $A^{-1} = A^T$

mtx mult:

$AB = C$

$m \times n \times n \times k = m \times k$

basically the same info

so what about $(AB)^T \neq C^T$\(\text{\times}\)\(\text{\times}\)\(\text{\times}\)\(\text{\times}\)

$AB \neq A^T B^T$

$m \times n \times n \times k = n \times m \times k \times n$

depends

so $(AB)^T = C^T = B^T A^T$

$k \times m \times k \times n = n \times m$

identity matrix

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$IA = A$

$Ix = x$

example of an orthogonal transformation:

$L x = 2 \times 2$ rotation matrix:

$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$

Want to show perpendicular columns:

$\vec{u} = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$

$\vec{v} = \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}$

Need to show 3 things:

1. are they perpendicular?

   take $\vec{u} \cdot \vec{v} = -\cos \theta \sin \theta + \sin \theta \cos \theta = 0$

   so we know $\vec{u} \perp \vec{v}$
(2) Check length

\[ ||\hat{u}|| = \sqrt{\hat{u} \cdot \hat{u}} = \sqrt{\cos^2 \theta + \sin^2 \theta} = 1 \text{ (trig!)} \]

\[ ||\hat{v}|| = 1 \text{ as well} \]

- note - simplest rotation matrix = \( I \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (\theta = 0) \)
- also: Improper Rotations are orthogonal transformations
  
  ex: \( A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \)

Today

- Orthogonal transformations cont.
- Rotations
- Rigid transformations
- Differential Drive

- checkout Linear Algebra Review Sheet and potential worksheet online
- everyone should be enrolled on Piazza
- Labs
  - self-scheduled
  - lab "kick-off" : n 30 minutes, introduction to labs
  - first lab next week

9/1/16
Properties of Orthogonal Transformations

* Preserve dot product.
  - If A is an orthogonal transform for any vectors \( \vec{x} \) & \( \vec{y} \):
    \[
    (A \vec{x}) \cdot (A \vec{y}) = \vec{x} \cdot \vec{y}
    \]
  - Last time: mentioned dot product tells us vectors are perpendicular.

Also
\[
\vec{u} \cdot \vec{v} = \cos \theta \|\vec{u}\| \|\vec{v}\|
\]

So dot product tells us how vectors are related (dot product \(\rightarrow\) vectors in diff. directions)

* Preserves length \( \|A \vec{x}\| = \|\vec{x}\| \)

* Closed under composition. If A & B are orthogonal transformations, so is AB
  \[
  AB \vec{x} = (AB) \vec{x} = A(B \vec{x})
  \]
  Let \( y = B \vec{x} \)
  \( z = A \vec{y} \)

* Transforms are applied from right to left

Ex: If you have 2 ortho. xforms
\[
\begin{bmatrix}
\cos \theta & \sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
-1 & 0 \\
0 & 1
\end{bmatrix} = \begin{bmatrix}\text{ortho. xform}\end{bmatrix}
\]

\[\text{or} \]
\[\text{Rotation} \quad \text{Reflection} \]

(Don't use these much)
Determinant

Measures the change in volume induced by a transformation.

\[ \text{det} A = 2 \]

if \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \text{det} A = ad - bc \)

so for:\n\[
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
-1 \\
0
\end{bmatrix}
\]
\[ \text{det} = \cos^2 \theta + \sin^2 \theta = 1 \]

\[ \text{det} = -1 \cdot 1 + 0 = -1 \]

A Rotation is an orthogonal transform whose determinant is 1.
\[ R^{-1} = R^T, \text{ det } R = 1 \]

A Rigid Transformation \( T(p) \) consists of:
- A Rotation Matrix \( R \)
- A Translation vector \( t \)

\[ T(p) = Rp + t = p' \]

Why do we care? Shows where stuff is & where it’s pointed.
turtlebot ex

(Robot view)

\[ y_w \]

\[ y_r \]

\[ x_r \]

\[ \theta \]

\[ x_w \] (World view)

\[ t_x \]

\[ t_y \]

\[ P_w = T^w_R (P_R) = R(\theta) P_R + t \]

\[ = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x_r \\ y_r \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \]

- Called rigid transformations because relative it preserves distance.

\[ || T(p_2) - T(p_1) || = || p_2 - p_1 || \]

(distance between points stays the same)

How can we invert transformation to get

\[ P_R = ? \]

We have \[ P_w = R(\theta) P_R + t \]

\[ \text{can't divide by } R(\theta) \text{, matrix math} \]

\[ P_w - t = R(\theta) P_R \]

\( \because \) because \( R(\theta) \) is a rotation \( \text{mtx} \)

\[ R(\theta)^T (P_w - t) = P_R \]

\[ \therefore \]

\[ P_R = \frac{R(\theta)^T P_w - R(\theta)^T t}{\text{rotation} \over \text{translation}} \]

Let's show that \( R(\theta)^T \) is same as \( R(-\theta) \)...
\[ R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \]  

\[ R(-\theta) = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \]

\[ \cos \theta : \quad \cos(-\theta) = \cos \theta \]

\[ \sin \theta : \quad \sin(-\theta) = -\sin \theta \]

*how to invert*

If \( T(p) \) is specified by \( R, t \), then

\( T^{-1}(p) \) is specified by \( R^T \) and \(-R^T t\)

*composition of rigid transformations*

\( T_1(p) = R_1 p + t_1 \)

\( T_2(p) = R_2 p + t_2 \)

\( T_2(T_1(p)) = R_2(R_1 p + t_1) + t_2 \)

\( = R_2 R_1 p + R_2 t_1 + t_2 \)

---

Is \( R_1 R_2 = R_2 R_1? \)

We know \( AB \neq BA \) !!!! (Question on HW)
Kinematics of Differential Drive

1. Two wheels on common axle can spin independently.

Turtlebot:

- Very bad 3D drawing

Passive rollers

\( \dot{x}_R = K r \)
\( \dot{y}_R = 0 \) (won't go sideways)
\( \dot{\theta}_R = 0 \) (no spinning)

What is the linear speed of point? 

Wheel
why no \( \pi \)?

angle \( \text{length} \)

\[ 2 \pi \text{ rad} \sim 2 \pi \text{ rad} \]

\[ 1 \text{ rad} \sim r \text{ m} \rightarrow \text{so just multiply by } \frac{1}{\text{rad}} \]

2. What if \( \text{v}_R = kr \), \( \text{v}_L = -kr \)

\[
\begin{align*}
\dot{x}_R &= 0 \text{ m/s} \\
\dot{y}_R &= 0 \text{ m/s} \\
\dot{\theta}_R &= ? \text{ rad/s} ^{1/s} \text{ same unit}
\end{align*}
\]

test: \( \dot{\theta}_R = k\text{ rad/s} \)

\[ \frac{1}{2} \text{ mm} - \text{m}^2/\text{s} \ldots \text{not the right unit!} \]

correct ans: \( \dot{\theta}_R = k \frac{r}{d} \)

Ways to get there?
- Think about proportionality
  - \( \dot{\theta}_R \propto kr \)
    - directly proportional to \( r \) because:

Think about circles
- like gears
- velocities must agree at point where they meet

\[
\begin{align*}
\dot{\theta}_R \text{ rad/s} &= kr \\
\dot{\theta}_R \text{ rad/s} &= \frac{kr}{d} \text{ rad/s}
\end{align*}
\]