In this worksheet, you will work out the equations of motion for a tricycle, and also discover why parallel front wheels are a bad idea for cars that need to turn.

The tricycle (see separate diagram) is a system with three wheels. The two rear wheels are parallel and separated by a distance $2\ell$. The front wheel sits ahead of the rear axle by a distance $d$. The tricycle is controlled in terms of its steering angle $\alpha$ and the angular velocity $\omega$ of its front wheel, which has radius $r$.

1. **Instantaneous Center of Curvature.** Find the ICC for the tricycle, and mark its location on the tricycle diagram. Remember, it will be the point which is at the center of the three concentric circles tangent to the wheels.

2. **Two key distances.** Define $h$ to be the length of line segment from the ICC to the center of the front wheel, and define $R$ to be the length of the line segment from the ICC to the midpoint of the rear axle. Solve for $h$ and $R$ in terms of the parameters $\ell$, $r$, $d$, $\alpha$, and $\omega$. Make sure you use the appropriate trig identities to reduce your solutions to their simplest form.

3. **Angular velocity.** Define $\theta_R$ to be the angular velocity of the tricycle about the ICC, and express it in terms of the parameters above. You can consider the front wheel and the circle of radius $h$ to be two “meshing gears”, if it helps.

4. **Linear velocity.** Define $\dot{x}_R$ to be the linear velocity of the middle of the rear axle of the tricycle as it travels around the circle of radius $R$, and express it in terms of the parameters above (again, simplify any trigonometric expressions you encounter).

5. **Can the tricycle turn in place?** That is, is there a setting for $\alpha$ such that the motion is purely angular, and not linear? If so, where is the ICC? If not, why?

6. **What’s wrong with this car?** Consider the car pictured on the back of this page. Why would it have trouble moving for any steering angle $\alpha \neq 0$?

7. **Why isn’t the tricycle here a valid kinematic system?** Formally, if we define the state as $q = (x, y, \theta)$ and the controls as $(\omega, \alpha)$, we don’t get a valid kinematic system as defined in class. How would including $\alpha$ in the state and introducing a new control $\dot{\alpha}$ (the derivative of the steering angle) fix the problem?