Exam Oct 1 in Class! Homeworks 1-4 covered today:
- Digital control of DC motors
- Configuration spaces
- Kinematic systems

**Problem #1** → How to vary motor speed when battery is outputting fixed voltage.

![PWM modulation diagram](image)

Your motor doesn't view this as "on and off" - Duty cycle = \( \frac{\text{Duty on}}{\text{Period}} \)

**Problem #2** → Motor only turns 1 direction

**Solution** → H-Bridge

Four computer controlled switches need an H-Bridge for each wheel.

**Convention** Motor spins forward when + Left - on Right

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<thead>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Result</th>
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<tbody>
<tr>
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<td>Forward</td>
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<td>Brake</td>
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<td>-</td>
<td>Free wheel</td>
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| ✓ |   | ✓ | - | - | Short Circuit → BAD!
**CONFIGURATION SPACES**

- A configuration \( q \) defines the state of a robot (pins down specifics all degrees of freedom) e.g. position/orientation

- A configuration space is the set of all configurations

  \[ X = \{ q \mid q \text{ is a robot state} \} \]

- For differential drive:

  \[ X = \{ q = (x, y, \theta) \mid x, y \text{ positions in m, } \theta \text{ angle in rad} \} \]

Is a configuration space the same as \( \mathbb{R}^n \)?

\[ q = (\theta_1, \theta_2) \]

**NO**, topology matters!

what configurations are "close"?

**COMMON TOPOLOGICAL SPACES IN ROBOTICS**

- \( \mathbb{R}^n \) - Good ol' cartesian coordinates
- \( S^2 \) - Unit circle (plane angle)
  - \( \theta \): radians
  - \( \theta \equiv \theta + 2\pi \equiv \theta + 4\pi \)

**special orthogonal group**

- \( \text{SO}(3) \) - Rotations in \( \mathbb{R}^3 \)

\( \mathbb{R}^2 \times S^2 \) - rigid xforms in 2D

\( \text{SE}(2) \)

\( \mathbb{R}^3 \times \text{SO}(3) \) - rigid xforms in 3D

\( \text{SE}(3) \)

**special euclidean groups**

\( \mathbb{R}^2 \times S^1 \)
A kinematic system is a mapping

\[ q_t = \sum_{i=1}^{m} V_i(q_t)w_i = \begin{bmatrix} V_1(a) & \ldots & V_m(a) \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_m \end{bmatrix} \]

where each \( V_i: \mathbb{R}^m \rightarrow \mathbb{R} \) maps a configuration to a velocity in configuration space, and each \( w_i \in \mathbb{R} \) is a control.

- \( V_i \)'s are functions (possibly non-linear).
- \( V_i \)'s depend on state.
- \( w_i \)'s combine to take a weighted sum (linear combination) of \( V_i \)'s.
- The result is a velocity in configuration space.
- Overall, this is a mapping from controls to velocities in configuration space.

For diff drive robot
\[ q_b = (x, y, \theta) \]
\[ \dot{q}_b = (\dot{x}, \dot{y}, \dot{\theta}) \]
\[ \theta = \frac{v_r - v_l}{2d} \]
\[ \dot{x} = \frac{v_r \cos\theta + v_l \cos\theta}{2} \]
\[ \dot{y} = \frac{v_r \sin\theta + v_l \sin\theta}{2} \]
\[ \dot{\theta} = \frac{v_r}{2d} \]

\[ \dot{q}_b = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \\
\dot{\theta} \\
\end{bmatrix} = \begin{bmatrix} \frac{v_r \cos\theta}{2} \\ \frac{v_r \sin\theta}{2} \\ \frac{v_l}{2d} \\ \frac{-v_l}{2d} \\
\end{bmatrix} \begin{bmatrix} v_r \\ v_l \\ \end{bmatrix} \]
KINEMATIC CONSTRAINTS

A constraint is an equation of the form

\[ f(\theta, \dot{\theta}) = 0 \]

eg for diff. drive system

linear velocity \((\dot{x}, \dot{y})\)

\[ f(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta}) = -\sin \theta \dot{x} + \cos \theta \dot{y} = 0 \]

\[ \text{there are places in a c-space you can't reach} \]
\[ \text{there are certain velocities in a c-space you can't achieve.} \]
Class Notes 9/24

Review from HW2: Write transformations from right to left

Implementation in code: ex:

c_from_a = c_from_b * b_from_a

b's agree

\[ T^L_A = \begin{bmatrix} 1 & 0 & -0.2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

Negative not positive number.

Sanity check: Pick a point in L and set what it is in A.

KINEMATIC CONSTRAINTS

\[ f(q, \dot{q}) = 0 \]

Review: \( q \) = configuration \( \dot{q} \) = configuration velocity

\( \rightarrow \) places (configurations) we can't reach

\( \rightarrow \) directions we can't travel in

\( \rightarrow \) think of a car parallel parking

\( \rightarrow \) we can derive them from kinematic system

\( \rightarrow \) sometimes we can derive system from constraints

Example:

\[ q = (x, y) \in \mathbb{R}^2 \]

\[ f(q, \dot{q}) = x^2 + y^2 - 1 = 0 \]

\[ x^2 + y^2 = 1 \]

ROBOT only gets to live in a radius of 1 away from the center.

Unit circle

HOLONOMIC VS. NON-HOLONOMIC CONSTRAINTS

\( \rightarrow \) Any constraint which may be written as \( f(q) = 0 \) is holonomic. "Places I can't go to"

\( \rightarrow \) If we can't, the constraint is non-holonomic

\( \rightarrow \) Is a constraint non-holonomic if a derivative appears in it?

\[ q = (x, y, \theta) \in \mathbb{R}^2 \times S^2 \]

\[ f_1(q, \dot{q}) = -x \sin \theta + y \cos \theta = 0 \]

\[ f_2(q, \dot{q}) = \dot{\theta} = 0 \]

At first, these appear as non-holonomic constraints. However, paired together they restrict the robot to traveling only in a straight line.
\[-(x-x_0)\sin\theta_0 + (y-y_0)\cos\theta_0 = C\]

\[\theta = \theta_0 \quad \text{substitute and integrate!}\]

\[\text{If you think you have a non-holonomic constraint, ask this:}\]

1. Can this vehicle get to any space it wants to in its environment?
2. Can I integrate a few things to eliminate velocities?

**KINEMATICS OF WHEELED VEHICLES**

- wheels don't go sideways (no slip condition)
- motion of wheeled vehicle valid only if all wheels traveling along concentric circles (or parallel lines)
- the circle's shared center is called the instantaneous center of curvature (ICC)

\[
\begin{align*}
R_1 &= R - d \\
R_2 &= R + d \\
\dot{\theta}(R-d) &= V_L r \\
\dot{\theta} &= \frac{V_L r}{R-d} \\
\dot{\theta}(R+d) &= V_R r \\
\dot{\theta} &= \frac{V_R r}{R+d}
\end{align*}
\]

\[
\begin{align*}
V_L &= \frac{V_R}{R-d} \\
V_{LR} &= \frac{V_R r}{R-d} \\
V_L &= \frac{V_R + V_L}{R} \\
R &= \frac{V_R + V_L}{V_R - V_L} \\
K &= \frac{1}{R} = \frac{1}{d} \frac{V_R - V_L}{V_R + V_L} \quad \text{curvature}
\end{align*}
\]