1. Bayes Filter

A clumsy, distractible, and somewhat illiterate robot is trying to determine whether it is in the West Wing or East Wing of Parish hall. The state $x \in \{\text{west}, \text{east}\}$ of the robot corresponds to its location. The robot may apply one of two actions $u \in \{\text{cross}, \text{stay}\}$, and subsequently try to observe whether it is standing outside the bank in the West Wing, or the Registrar’s office in the East Wing, with $z \in \{\text{bank}, \text{registrar}\}$.

The robot is not too bad at crossing from the West to the East, with a failure rate of just 5%; however when crossing from East to West, the robot has a 30% chance of becoming distracted by a student playing piano in the parlor, and wandering back to where it started. When attempting to stay put, the robot has a 10% failure rate in the West Wing (owing, again, to the allure of the piano); however it has a perfect 100% success rate in staying put in the East Wing.

When in the West Wing, the robot is pretty good at recognizing the bank, with a failure rate of just 5%; however, the robot often mistakes the Registrar’s office in the East Wing for a bank as well, 15% of the time.

a. Write down the motion model of the robot as a list or table of eight conditional probabilities of the form $p(x'|x, u)$.

b. Write down the sensor model of the robot as a list or table of four conditional probabilities of the form $p(z|x)$.

c. Initially, the robot believes there is a 50% chance that it is in either wing (i.e. $\text{bel(}west\text{)} = \text{bel(}east\text{)} = 0.5$). It then applies the action $u = \text{cross}$. After the motion update, what is $\text{bel}(x)$ for each state?

d. Subsequently, the robot makes an observation of $z = \text{bank}$. After the measurement update, what is $\text{bel}(x)$ for each state?