1. Tent-shaped continuous probability distribution

We will design a program to sample from a tent-shaped probability density function, as pictured below:

![Tent-shaped PDF diagram](image)

Note that \( p(x) = 0 \) for \( x \notin [0, 1] \).

a. What must the height \( h \) of the tent be, and why? Use your answer to derive a piecewise-linear function defining the probability density function \( p(x) \) for the domain \( x \in [0, 1] \).

b. Derive a piecewise quadratic cumulative distribution function \( cdf(x) \) for the domain \( x \in [0, 1] \).

c. Finally, derive the inverse function \( cdf^{-1}(q) \) for \( q \in [0, 1] \).

d. Write a program to plot \( p(x) \), \( cdf(x) \), and \( cdf^{-1}(q) \) over the unit interval.

e. Write a program to sample from \( p(x) \) by mapping a uniformly generated random number \( q \in [0, 1] \) through the inverse distribution function. Demonstrate the effectiveness of your method by generating a histogram showing the frequencies of 10,000 total samples in 20 equally spaced bins over the unit interval (i.e. each with a width of 0.05).

Submit printouts of both program sources along with clearly labeled plots.