Today
- Map Representations
- Navigation
- Dijkstra's Alg.

Navigation
Given some representation of free space, conduct robot safely from start to goal.

Map Representations

```
START + Room

Table

GOAL

```

How do we represent the room?
Discretize?

```
Room

← "Occupancy grid"

```

This thing?
Room

```
Room

← "Navigation Mesh"

← "Convex Cell Decomposition"

```

Convex?
Yes

No
Means you can't draw this line
Can represent these representations using graphs for the Navigation Mesh example, we have:

```
A
/ \ /  \
B   F
\ /  \
C   E
```

For Occupancy Grid:

```
1 1 1 1 1 1 1 1
1 0 1 0 1 0 1 1
1 1 1 1 1 0 0 1
1 0 1 0 1 0 1 1
1 1 1 1 1 1 1 1
```

Q: How do we navigate from start to finish in these representations?
A: Something like Dijkstra's Alg. will work!

**Dijkstra's Algorithm**
- Check out the worksheet!

**A**<sup>*</sup> - Alteration with Heuristics

- Initialize \( g(x) = \infty \) for all \( x \)
- Initialize \( \text{pred}(x) = \emptyset \) for all \( x \)
- Initialize \( g(x_{\text{init}}) = 0 \)
- Initialize \( Q = \{ x_{\text{init}} \} \)

While \( Q \) not empty
- Extract \( x \) from \( Q \) with \( g(x) + h(x) \)

// same as Dijkstra's

end
$h(x)$ in A*
- Optimistic Estimator of Cost-to-go
- We need $h(x)$ to be a Lower Bound on true optimal cost-to-go

If $h(x) = 0$?
  - We have Dijkstra's

If $h(x)$ is (somehow) actual true optimal C2G
  - A* finds path in linear time! (Relative to Path)
Today

- Occupancy Grid Details
- Probability Basics

Last time: World Representations where we can traverse / Navigate using graph search

Occupancy Grid Details

Questions:
- How big should cells be?
  - Too big - Miss Narrow Passageways
  - Too small - Memory Required goes up
    - Long Planning Times

Represent 2F of Hicks as an occupancy grid for a turtlebot:
- 1 Cell: 25 cm x 25 cm
- Could go bigger - Most features in
  - Hicks are bigger
- Could go smaller

Estimate cost for this resolution:
Say Hicks 2F is 30 x 20 m
  \[ 3000 \text{ m}^2 \]
  \[ 16,000 \text{ cells @ 25 cm}^2 \]
  \[ 400,000 \text{ cells @ 5 cm}^2 \]

at 1 bit / cell = 21 kB @ 25 cm$^2$
  \[ 50 kB @ 5 \text{ cm}^2 \]
Uncertainty
- Smart Robots Deal with Uncertainty
- Maintain Probabilistic Quantities

Probability Basics
+ $X$ is a Random Variable if it can take on one of a number of values in a set

$$X = \{x_1, x_2, \ldots, x_n\}$$

+ The Probability $P(X = x_i)$ that a variable $(X)$ takes a value $x_i$ is the expected frequency of obtaining the result $(x_i)$ in a set of independent trials

The degree of belief that the next observation of $X$ will be $x_i$

EX: we have 90 humans, 10 cylons in a spaceship - pick a random person $X$

$$X = \{\text{human}, \text{cylon}\}$$

$p(X = \text{human}) = 0.9$

$p(X = \text{cylon}) = 0.1$

Properties

$$\sum_{i=1}^{n} P(X = x_i) = 1$$

$p(X = x_i) \geq 0$ for all $x_i$

$p(X = x_i) \leq 1$ for all $x_i$

Laziness

$p(\text{human}) \Rightarrow p(X = \text{human})$
Joint Distributions

\( P(x, y) \) is the probability that \( x = x \) and \( y = y \) simultaneously.

Two Random Variables are Independent of

\[ P(x, y) = P(x) P(y) \]

E.g., Rolling Double 6's

\[ P(\text{Die 1} = 6, \text{Die 2} = 6) = P(\text{Die 1} = 6) \cdot P(\text{Die 2} = 6) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \]

Conditioning

\( P(x | y) \) is probability that \( x = x \) given that we already observed \( y = y \)

\[ P(x | y) = \frac{P(x, y)}{P(y)} \]

If \( x | y \) independent

\[ P(x | y) = P(x) \]

Example: Someone develops a Cylon Test

\[ P(\text{positive} | \text{Cylon}) = 0.80 \]

\[ P(\text{negative} | \text{Cylon}) = 0.20 \quad \text{False Neg} \]

\[ P(\text{positive} | \text{Human}) = 0.1 \quad \text{False Pos} \]

\[ P(\text{negative} | \text{Human}) = 0.9 \]

Theorem of Total Probability

Bayes Rule

\[ P(x) = \sum_y P(x | y) P(y) \]

\[ P(x | y) = \frac{P(y | x) P(x)}{P(y)} \]

\[ P(x | y) = \frac{P(y | x) P(x)}{\sum_x P(y | x') P(x')} \]
Back to example...

What is \( P(\text{positive}) \)?

\[
P(\text{pos}) = P(\text{pos} \mid \text{cylon}) \cdot P(\text{cylon}) + P(\text{pos} \mid \text{human}) \cdot P(\text{human})
= 0.8 \cdot 0.1 + 0.1 \cdot 0.9
= 0.08 + 0.09 = 0.17
\]

Q: Should I throw someone out the airlock??

Instead ask \( P(\text{cylon} \mid \text{positive}) \)?

**How? Bayes Rule**

\[
P(\text{cylon} \mid \text{positive}) = \frac{P(\text{positive} \mid \text{cylon}) \cdot P(\text{cylon})}{P(\text{positive})}
= \frac{0.8 \cdot 0.1}{0.17}
= \frac{0.08}{0.17} \approx 0.47\%
\]

Think about it this way:

100 people

\[\begin{array}{c}
\text{9 human, 10 cylons} \\
\text{9 pos, 81 neg} \quad \text{8 pos, 2 neg}
\end{array}\]

For our actual stuff, we actually want to know

\( P(\text{Location} \mid \text{sensor reading}) \)

But easier to form

\( P(\text{sensor} \mid \text{location}) \)