1. Epipolar geometry and pure rotation

Consider the multiple view geometry when two cameras’ poses are related by a pure rotation \( R \), with zero translation \( t = 0 \). You may assume that both cameras have an identity intrinsic parameter matrix \( K = I \), so that the normalized image plane and the sensor plane are coincident.

   a. Explain why the essential matrix becomes degenerate in this case.

   b. Consider a 3D point \( P \) which is mapped by the two camera projection matrices to the points \( q \) and \( q' \), respectively. Show that the projections in this rotation-only case can be related by a homography \( q' \equiv Hq \). What is \( H \)?

2. Fitting fundamental matrices from data

Download fundamental.zip and run the fundamental.py script on each pair of images in the archive. When you hover your mouse over the left image of the demo, you will see the corresponding epipolar line in the right image, and vice versa.

   a. Which pair of images (book or food) is set up more like a traditional stereo arrangement (pure sideways camera translation, no rotation)? How can you tell this from the configuration of the epipolar lines?

   b. The epipoles in both image planes are the points where all epipolar lines intersect. For which pair of images (book or food) do the epipoles lie closest to the image centers, and how can you tell?

   c. How would you describe the epipoles in an ideal stereo camera setup? Where on the image plane do they lie?

   d. Type up a paragraph or two explaining how the fundamental.py script generates possible point correspondences between the two images. Your answer should define the terms keypoint and descriptor and explain their relevance to this problem. **Hint:** You may find sections 4.1.1 and 4.1.2 of the Szeliski textbook helpful background reading, as well as the OpenCV documentation.
3. Orthographic projection

Imagine a camera whose projection matrix is defined as:

\[
M_k = K[R \mid t] = \begin{bmatrix}
kf & 0 & 0 \\
0 & kf & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\mathbf{r}_1^T & t_x \\
\mathbf{r}_2^T & t_y \\
\mathbf{r}_3^T & t_z + k
\end{bmatrix}
\]

Note that as \(k\) increases, we increase both the focal length of the camera, and the \(Z\) distance between the camera and some point in the world by the same factor.

Let’s consider the projection of some point \(P = (X, Y, Z)\). We have

\[
\begin{bmatrix}
x \\
y \\
w
\end{bmatrix} = \begin{bmatrix}
kf & 0 & 0 \\
0 & kf & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\mathbf{r}_1^T & t_x \\
\mathbf{r}_2^T & t_y \\
\mathbf{r}_3^T & t_z + k
\end{bmatrix}
\begin{bmatrix}
P \\
1
\end{bmatrix}
\]

\[
= \begin{bmatrix}
kf(\mathbf{r}_1^T P + t_x) \\
kf(\mathbf{r}_2^T P + t_y) \\
\mathbf{r}_3^T P + k + t_z
\end{bmatrix}
\]

a. If we use perspective division to obtain \((u, v, 1) \equiv (x, y, w)\), what are \(u\) and \(v\) in terms of \(P\) and the elements of \(M_k\)?

b. Take the limit of \(u\) and \(v\) as \(k\) goes to infinity, and use L’Hôpital’s rule to express each limit as simply as possible.

c. Construct a matrix \(M_\infty\) that has the same effect of \(M_k\) above, as \(k \to \infty\). That is, we want the matrix to satisfy

\[
\begin{bmatrix}
\lim_{k \to \infty} u \\
\lim_{k \to \infty} v \\
1
\end{bmatrix} = M_\infty \begin{bmatrix}
P \\
1
\end{bmatrix}
\]

d. The resulting projection is called an orthographic projection. Explain why this projection is kind of like an “ultra-telephoto” lens. Hint: You may find it helpful to browse this Wikipedia article: http://en.wikipedia.org/wiki/Perspective_distortion_%28photography%29