1. Epipolar geometry

Let’s examine two special cases of epipolar geometry. Assume you have two cameras with trivial intrinsic parameter matrices $K$ equivalent to the identity matrix (so that the normalized image plane and sensor plane are coincident). Also, assume that the cameras are oriented in the same direction (so that $R$ is also the identity matrix).

Recall that the epipole in each camera’s image plane is just the projection of the other camera’s optical center in the image. Note that the epipole is where the set of all epipolar lines intersect.

a. Epipoles for translation along the optical axis. Assume that the second camera view is obtained by translating straight forward along the optical axis of the first camera (i.e., walking directly forwards). Describe where the epipoles in the two images lie.

b. Epipoles for translation perpendicular to the optical axis. Now, imagine that the second camera view is obtained by translation perpendicular to the optical axis of the first camera (i.e., walking directly sideways). Do the epipoles lie within the cameras’ fields of view? What is the configuration of the epipolar lines in this case?

2. Pure rotation

Consider the multiple view geometry when cameras are related by a pure rotation $R$, with zero translation $t = 0$. Again, you may assume that both cameras have an identity intrinsic parameter matrix $K = I$.

a. What happens to the essential matrix?

b. Show that the two images can be related by a homography such that $q' = Hq$. Explain how to derive $H$ solely in terms of the relative transformation between the cameras (i.e., irrespective of any particular points $q, q'$).
3. Orthographic projection

Imagine a camera whose projection matrix is defined as:

\[
M_k = K[R \mid t] = \begin{bmatrix}
  k f & 0 & 0 \\
  0 & k f & 0 \\
  0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
  r_1^T & t_x \\
  r_2^T & t_y \\
  r_3^T & t_z + k \\
\end{bmatrix}
\]

Note that as \( k \) increases, we increase both the focal length of the camera, and the \( Z \) distance between the camera and some point in the world by the same factor.

Let’s consider the projection of some point \( P = (X, Y, Z) \). We have

\[
\begin{bmatrix}
  x \\
  y \\
  w
\end{bmatrix} = \begin{bmatrix}
  k f & 0 & 0 \\
  0 & k f & 0 \\
  0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
  r_1^T & t_x \\
  r_2^T & t_y \\
  r_3^T & t_z + k \\
\end{bmatrix}
\begin{bmatrix}
P \\
1
\end{bmatrix}
\]

\[
= \begin{bmatrix}
k f(r_1^T P + t_x) \\
k f(r_2^T P + t_y) \\
r_3^T P + k + t_z
\end{bmatrix}
\]

a. Where on the normalized image plane does the point \( P \) project to?

b. Take the limit of \( x/w \) and \( y/w \) as \( k \) goes to infinity. Does the limit exist?

c. Construct a matrix \( M_\infty \) that has the same effect of \( M_k \) above, with \( k = \infty \).

d. The resulting projection is called an orthographic projection. Explain why this projection is kind of like an “ultra-telephoto” lens. Hint: You may find it helpful to browse this Wikipedia article: http://en.wikipedia.org/wiki/Perspective_distortion_%28photography%29