ENGINEERING 12: PHYSICAL SYSTEMS ANALYSIS
LABORATORY 4
BIOPHYSICS AND CURVE FITTING

OBJECTIVES

1. To understand the parameterization of second order systems in terms of natural frequency $\omega_0$ and damping ratio $\zeta$.

2. To gain experience in characterizing physical systems via curve fitting.

APPARATUS

Goniometer, 1.5 kg weight, oscilloscope, PC equipped with MATLAB.

INTRODUCTION

In this experiment, we will attempt to characterize the step response of the human forearm as a second order system. Each group will collect timeseries data of elbow angle as the arm is bent from an initial straight configuration to an elbow angle of 90°. The resulting data traces will be analyzed in order to determine the natural frequency and damping ratio of the biophysical system.

THEORY

At first glance, second order systems appear to be determined by three parameters. For instance, an RLC circuit obeys the equation

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$

and a damped mass-spring system follows

$$m\ddot{x} + B\dot{x} + Kx = 0$$

However, the dynamics are identical if we multiply the left hand side of either equation above by an arbitrary constant (for instance, doubling the mass, stiffness and damping coefficients above). Hence, although there are three parameters describing each of these systems, there are only two degrees of freedom for the dynamics of a second order system.

We can choose those degrees of freedom to be $\omega_0$ and $\zeta$, the natural frequency or undamped frequency of the system, and the damping ratio, respectively. The corresponding dynamics are given by

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2x = 0$$

The natural frequency $\omega_0$ is measured in radians per second, and is the frequency at which the system would oscillate if it were undamped. The damping ratio $\zeta$ is a unitless quantity that determines
whether the system is overdamped ($\zeta > 1$), critically damped ($\zeta = 1$), or underdamped ($\zeta < 1$). In the case of an underdamped system, the solution will oscillate at a frequency of $\omega_d = \omega_0 \sqrt{1 - \zeta^2}$.

For the damped mass-spring system above, it can be shown that

$$\omega_0 = \sqrt{\frac{K}{m}}$$

and

$$\zeta = \frac{B}{2\sqrt{Km}}$$

**PROCEDURE**

1. Begin by turning on the goniometer and firmly strapping onto an arm so that the hinge point is coincident with the elbow.

2. Record four traces of data: bending the arm to $90^\circ$ for each of two individuals, both with and without the hand holding a 1.5 kg weight. Make sure that the position converges to a constant value at the end of each trace.

3. When you are finished collecting data, turn off the goniometer.

4. Save the data as spreadsheet format, and bring it into MATLAB for analysis.

**ANALYSIS**

Write a function that uses your fourth order Runge-Kutta code from the last lab to simulate a generic second order system. The function should be of the form

$$function \ x = \text{second\_order}(t, \ x0, \ dx0, \ w0, \ z)$$

where $t$ is an $N$-by-1 vector of times, $x0$ is the initial position of the system, $dx0$ is the initial velocity, and $w0$ and $z$ are the natural frequency and damping ratio, respectively. The function should return $x$, an $N$-by-1 vector of positions.

Next, for each timeseries, subtract off the vertical offset so that the system converges to the zero position at the end of each trace. Try to estimate $x0$, $dx0$, $\omega0$, and $\zeta$ by plotting your estimate against the data, using the `second\_order` function you wrote. You don’t need to get the parameters exactly right, but you need a decent initial guess in order to run the automatic fitting procedure below.

Finally, copy and paste the script below to fit the data exactly. You will want to set the variable `pinit` to a 4-vector containing the initial guess for position, velocity, natural frequency, and damping ratio, and the variables $t$ and `xmeas` should contain the times and measurements from the scope.
When \texttt{fminsearch} returns, \texttt{pfit} should contain the best-fit estimates of the parameters.

\begin{verbatim}
  sim = @(p) second_order(t, p(1), p(2), p(3), p(4));
  err = @(p) sum((sim(p) - xmeas).^2);
  pfit = fminsearch(err, pinit);
\end{verbatim}

\textbf{LABORATORY REPORT}

1. In your theory section, derive the natural frequency and damping ratio for the RLC system, and use dimensional analysis to show that $\omega_0$ can be measured in rad/s, and $\zeta$ is dimensionless for both systems.

2. Look up \texttt{fminsearch} in the MATLAB documentation. What is it doing, and what does each line of the small script above accomplish?

3. Explain why the weight changes the natural frequency and damping ratio for each individual. Did the weight have a similar effect on both individuals?

4. Include plots of the captured data, and best-fit curves for each individual and condition (with and without weight).

5. To what extent is it reasonable to describe this physical system (i.e. the human arm) as a single second order system with two parameters? What are the benefits and drawbacks of considering the system in this manner?