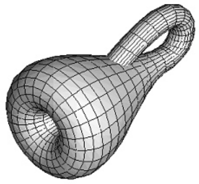

THIRD ANNUAL
SWARTHMORE COLLEGE
PROBLEM-SOLVING COMPETITION
Sunday, November 16th 2008

Problem 1

Write the following sum as a integer:

$$\frac{1}{\sqrt{1}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{9}}+\ldots+\frac{1}{\sqrt{10199}+\sqrt{10201}}$$

Problem 2

Find all solutions in positive integers to the equation $n!+3=m^2$.

Problem 3

You are blindfolded and are given a row of x pennies in front of you. You are told that y of them are heads up. You are unable to tell, by touch, which side of the penny is heads or tails. Describe how to place all the pennies into two groups so that each group has an equal number of pennies that are heads up. (You may blindly turn over any pennies you wish.)

Problem 4

Consider all sequences $a_1, a_2, \ldots$ that can be constructed recursively as follows: Set $a_1 = 1$ and for all $n > 1$, pick a positive integer $k_n < n$ and then choose a_n so that

$$a_{k_n} + a_{k_n+1} + \cdots + a_{n-1} + a_n = 0.$$

Let S be the set of all numbers which occur in at least one of the sequences constructible in this way. Determine, with proof, all members of S.

Problem 5

Let a, b, c be real numbers, and let $f(x) = x^3 + ax^2 + bx + c$. Now define a polynomial g by $g(x) = 2f''(x)f(x) - f'(x)^2$. Then g has degree 4. Show that g DOES NOT have four distinct real roots.

Problem 6

Each of the 21 dots in a 3×7 array is colored blue or green. Show there must be four dots of the same color that form the corners of a rectangle whose base is horizontal.