Cascade Compensation

- Design via s-plane
- Transform controller into z-plane
  - Transformation that preserves behavior of continuous compensator
- Tustin Transformations:
  - Bilinear transformation that yields digital transfer function whose output matches analog version at the sampling instants

$$s = \frac{2}{T}\frac{z-1}{z+1}$$

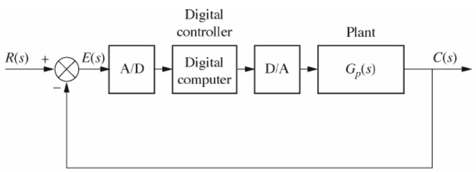
$$z = \frac{1+\frac{T}{2}s}{1-\frac{T}{2}s}$$

Choosing T

- If T is too large (or too low sampling frequency)
- In general, upper bound on T should be

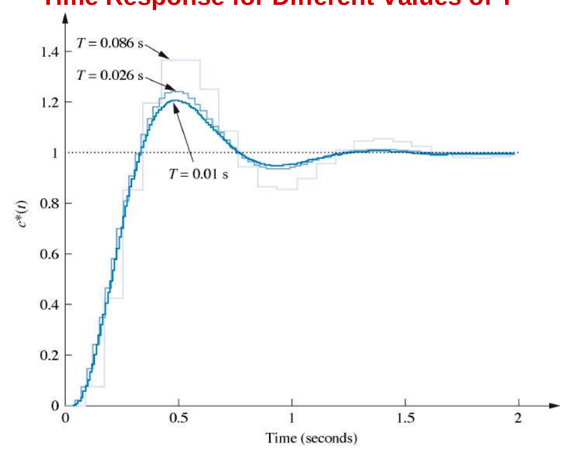
$$\frac{0.15}{\omega_{\Phi_M}} \leq T \leq \frac{0.5}{\omega_{\Phi_M}}$$

Example



- Given $G_p(s)$ = 1/[s(s+6)(s+10)], design lead controller s.t. system will operate at 20% overshoot
- Assume our lead controller is $G_c(s) = \frac{1977(s+6)}{s+29.1}$

Time Response for Different Values of T



Note: Valid only at integer values of sampling instant

Example

Let $G_p(s) = 100K/[s(s+3)(s+100)]$, design lead compensator for 20% overshoot, $T_p = 0.1$ sec and $K_v = 40$.

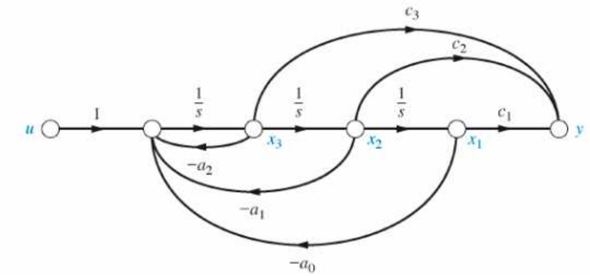
For the given design specs, K = 1440 and

$$G_c(s) = 2.38\frac{s + 25.3}{s + 60.2}$$

Pole Placement

- Given $G(s) = 20(s+5)/[s(s+1)(s+4)]$, design phase-variable feedback gains to yield 9.5% overshoot w/ $T_s = 0.74$ sec

- Locations of poles: -5.4±j7.2, -5.1



Controllability

If an input to a system can be found that takes every state variable from a desired initial state to a desired final state, the system is said to be *controllable*; otherwise, the system is *uncontrollable*.