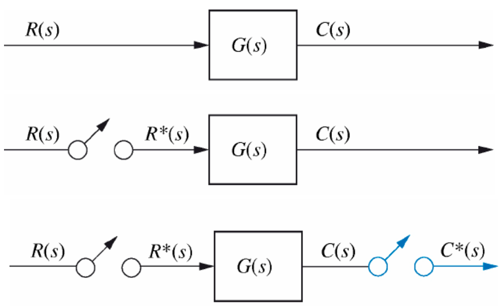
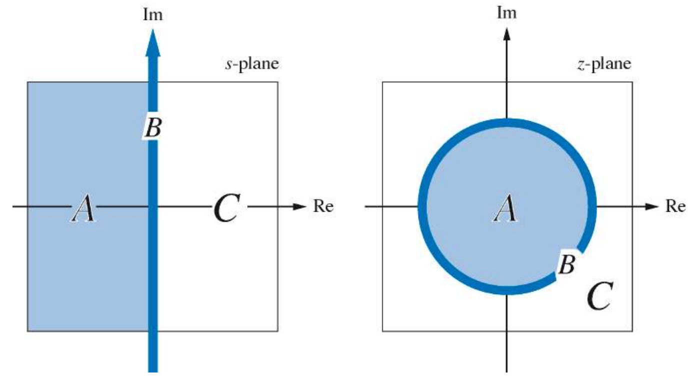
Transfer Functions



$$C(z) = G(z)R(z)$$

Stability



Stability via the s-Plane

- Routh-Hurwitz criterion for stability
- More helpful to have transformations that are linear
  - Bilinear Transformations between s-Plane and z-Plane

$$s = \frac{z+1}{z-1}$$
$$z = \frac{s+1}{s-1}$$

Example

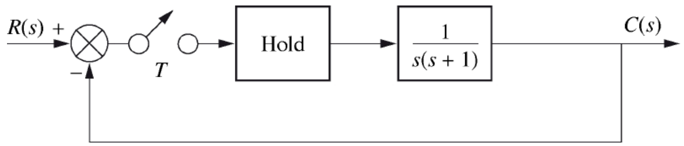
Given T(z) = N(z)/D(z) s.t. D(z) = $z^3 - z^2 - 0.5z + 0.3$, determine number of z-plane poles of T(z) inside, outside, and on the unit circle.

Solution:

1. Substitute z = (s+1)/(s-1) into D(z) = 0
2. Generate Routh Table for equivalent D(s) = 0
3. Number of sign changes gives # of poles outside unit circle

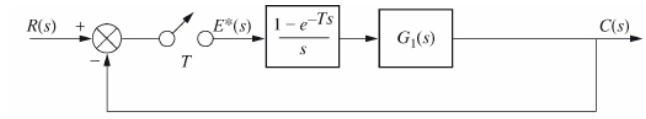
T Affects Stability

What is the range of T that will keep system stable?



Characterizing System Performance in z-Plane

- Placement of sampler changes open-loop transfer function
- Assumption:



Steady-State Error in the z-Plane

In the s-plane

$$K_p = \lim_{s \to 0} G(s)$$

$$K_v = \lim_{s \to 0} sG(s)$$

$$K_a = \lim_{s \to 0} s^2 G(s)$$

In the z-plane

$$K_p = \lim_{z \to 1} G(z)$$

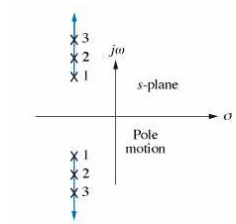
$$K_v = \frac{1}{T} \lim_{z \to 1} (z-1)G(z)$$

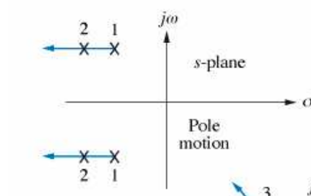
$$K_a = \frac{1}{T^2} \lim_{z \to 1} (z-1)^2 G(z)$$

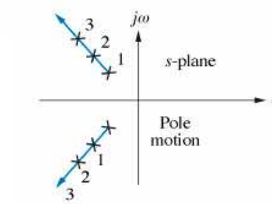
Transient Performance Specifications

In the s-plane

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d}$$







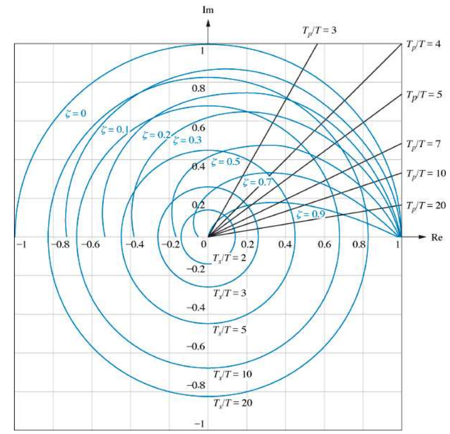
$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{\sigma_d}$$

$$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%$$

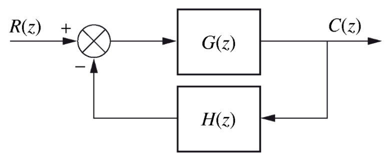
Example

Find K_p, K_v and K_a error for the feedback control system w/ $G_1(s) = 20(s+3)/[(s+4)(s+5)]$

In the z-Plane

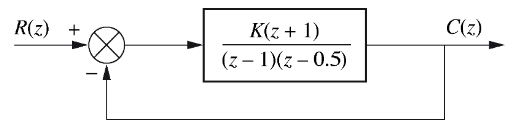


Gain Design



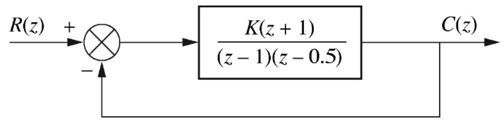
- z-Plane vs. s-Plane
  - Root Locus in the z-Plane

Example



- What range of K is system stable?

Example



- What K yields ζ = 0.7?