Honors Multivariate Calculus Placement Problems

Instructions. These problems are for math students who have studied theoretical multivariate calculus before entering Swarthmore and want to know if they can be placed out of our spring Honors course Math 35. (Most entering students interested in advanced mathematics take 35 even if they have had regular multivariate calculus before.)

You may not have any notes or textbooks open while working on these problems. You are welcome to use mathematical software on computers or calculators, but always indicate where and how you have used them.

Put your name and "Honors Multivariate Calculus Placement" on each page of your solutions. Mail to the Department of Mathematics and Statistics, 500 College Ave, Swarthmore PA 19081-1390, by August 15. You need not return this question sheet. If you have not placed out of a full year of first-year calculus based on standardized tests, please submit your answers to the problems below *along with* Swarthmore's Calculus Placement Exam, not instead.

1. Let $f(x, y) = xye^{-x-y}$.

a) Find all the critical points and show that there is just one local extremum.

b) Show that this local extremum is not a global extremum.

Note: You may be able to do this problem by first-year calculus methods, but don't. Show us that you know multivariate calculus methods.

2. The unit sphere S starts with its center at the origin at time $t = 0$ and moves (without rotating) so that its center is at (t, t, t) at time t. Suppose the temperature at point (x, y, z) in space is $T = x + y^2 + 2z$.

a) At time $t = 0$, what point (or points) on S is hottest?

b) At time $t = 0$, at what point on S is the temperature increasing fastest (due to S moving)?

3. Using triple integration in some coordinate system, determine the volume of the ball of radius 1 centered at the origin in $\mathbb{R}^3$.

4. By a method of your choosing, determine the hypervolume and hyper surface area of the hyperball of radius 1 centered at the origin in $\mathbb{R}^4$.

5. Let S be the upper hemisphere of the sphere of radius 3 centered at $(0, 0, 0)$. That is,

$$S = \{(x, y, z) \mid x^2 + y^2 + z^2 = 9,\ z \geq 0\}.$$

Let $\vec{F} = (x, y, z)$. Compute

$$\iint_S \vec{F} \cdot d\vec{S}.$$

Compute this integral directly. If you know a theorem or an insight that allows for an easier computation, show that second.

6. Let curve C be the boundary of some surface S in R^3. Let $f, g : R^3 \to R$ be differentiable. By the general Stokes Theorem,

$$\int_C f\,dx + g\,dy = \int_S A\,dx \wedge dy + B\,dy \wedge dz + C\,dz \wedge dx.$$

What are A, B, C?