MAXWELL'S EQUATIONS

Integral	*Differential*	*Boundary Conditions*
Faraday's Law		
$\oint_L \mathbf{E}' \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\mathbf{n} \times (\mathbf{E}_2' - \mathbf{E}_1') = 0.$
Ampere's Law with Maxwell's Displacement Current Correction		
$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J}_f \cdot d\mathbf{S} + \frac{d}{dt}\int_S \mathbf{D} \cdot d\mathbf{S}$	$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$	$\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}_f$
Gauss's Law		
$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho_f dV$	$\nabla \cdot \mathbf{D} = \rho_f$	$\mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \sigma_f$
$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$	$\nabla \cdot \mathbf{B} = 0$	$\mathbf{n} \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0$
Conservation of Charge		
$\oint_S \mathbf{J}_f \cdot d\mathbf{S} + \frac{d}{dt}\int_V \rho_f dV = 0$	$\nabla \cdot \mathbf{J}_f + \frac{\partial \rho_f}{\partial t} = 0$	$\mathbf{n} \cdot (\mathbf{J}_2 - \mathbf{J}_1) + \frac{\partial \sigma_f}{\partial t} = 0$

Usual Linear Constitutive Laws

$\mathbf{D} = \varepsilon \mathbf{E}$

$\mathbf{B} = \mu \mathbf{H}$

$\mathbf{J}_f = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = \sigma \mathbf{E}'$ [Ohm's law for moving media with velocity $\mathbf{v}$]

PHYSICAL CONSTANTS

Constant	Symbol	Value	units
Speed of light in vacuum	c	$2.9979 \times 10^8 \approx 3 \times 10^8$	m/sec
Elementary electron charge	e	1.602×10^{-19}	coul
Electron rest mass	m_e	9.11×10^{-31}	kg
Electron charge to mass ratio	$\frac{e}{m_e}$	1.76×10^{11}	coul/kg
Proton rest mass	m_p	1.67×10^{-27}	kg
Boltzmann constant	k	1.38×10^{-23}	joule/°K
Gravitation constant	G	6.67×10^{-11}	nt-m^2/(kg)^2
Acceleration of gravity	g	9.807	m/(sec)^2
Permittivity of free space	ε_0	$8.854 \times 10^{-12} \approx \frac{10^{-9}}{36\pi}$	farad/m
Permeability of free space	μ_0	$4\pi \times 10^{-7}$	henry/m
Planck's constant	h	6.6256×10^{-34}	joule-sec
Impedance of free space	$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$	$376.73 \approx 120\pi$	ohms
Avogadro's number	N	6.023×10^{23}	atoms/mole

VECTOR IDENTITIES

$(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{C} \times \mathbf{A}) \cdot \mathbf{B}$

$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

$\nabla \cdot (\nabla \times \mathbf{A}) = 0$

$\nabla \times (\nabla f) = 0$

$\nabla (fg) = f \nabla g + g \nabla f$

$\nabla (\mathbf{A} \cdot \mathbf{B}) = (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A})$

$\nabla \cdot (f\mathbf{A}) = f \nabla \cdot \mathbf{A} + (\mathbf{A} \cdot \nabla) f$

$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

$\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$

$\nabla \times (f\mathbf{A}) = \nabla f \times \mathbf{A} + f \nabla \times \mathbf{A}$

$(\nabla \times \mathbf{A}) \times \mathbf{A} = (\mathbf{A} \cdot \nabla)\mathbf{A} - \frac{1}{2}\nabla(\mathbf{A} \cdot \mathbf{A})$

$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

INTEGRAL THEOREMS

Line Integral of a Gradient

$$\int_a^b \nabla f \cdot d\mathbf{l} = f(b) - f(a)$$

Divergence Theorem:

$$\int_V \nabla \cdot \mathbf{A} dV = \oint_S \mathbf{A} \cdot d\mathbf{S}$$

Corollaries

$$\int_V \nabla f dV = \oint_S f d\mathbf{S}$$

$$\int_V \nabla \times \mathbf{A} dV = -\oint_S \mathbf{A} \times d\mathbf{S}$$

Stokes' Theorem:

$$\oint_L \mathbf{A} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S}$$

Corollary

$$\oint_L f d\mathbf{l} = -\int_S \nabla f \times d\mathbf{S}$$

FROM ZAHN,
ELECTROMAGNETIC
FIELD THEORY

Cartesian Coordinates (x, y, z)

$$\nabla f=\frac{\partial f}{\partial x}\mathbf{i}_x+\frac{\partial f}{\partial y}\mathbf{i}_y+\frac{\partial f}{\partial z}\mathbf{i}_z$$

$$\nabla\cdot\mathbf{A}=\frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A}=\mathbf{i}_x\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)+\mathbf{i}_y\left(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right)+\mathbf{i}_z\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 f=\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}$$

Cylindrical Coordinates (r, ϕ, z)

$$\nabla f=\frac{\partial f}{\partial \mathrm{r}}\mathbf{i}_r+\frac{1}{\mathrm{r}}\frac{\partial f}{\partial \phi}\mathbf{i}_\phi+\frac{\partial f}{\partial z}\mathbf{i}_z$$

$$\nabla\cdot\mathbf{A}=\frac{1}{\mathrm{r}}\frac{\partial}{\partial \mathrm{r}}(\mathrm{r}A_r)+\frac{1}{\mathrm{r}}\frac{\partial A_\phi}{\partial \phi}+\frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A}=\mathbf{i}_r\left(\frac{1}{\mathrm{r}}\frac{\partial A_z}{\partial \phi}-\frac{\partial A_\phi}{\partial z}\right)+\mathbf{i}_\phi\left(\frac{\partial A_r}{\partial z}-\frac{\partial A_z}{\partial \mathrm{r}}\right)+\mathbf{i}_z\frac{1}{\mathrm{r}}\left[\frac{\partial(\mathrm{r}A_\phi)}{\partial \mathrm{r}}-\frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 f=\frac{1}{\mathrm{r}}\frac{\partial}{\partial \mathrm{r}}\left(\mathrm{r}\frac{\partial f}{\partial \mathrm{r}}\right)+\frac{1}{\mathrm{r}^2}\frac{\partial^2 f}{\partial \phi^2}+\frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinates (r, θ, ϕ)

$$\nabla f=\frac{\partial f}{\partial r}\mathbf{i}_r+\frac{1}{r}\frac{\partial f}{\partial \theta}\mathbf{i}_\theta+\frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\mathbf{i}_\phi$$

$$\nabla\cdot\mathbf{A}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r)+\frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{A}=\mathbf{i}_r\frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta A_\phi)}{\partial\theta}-\frac{\partial A_\theta}{\partial\phi}\right]+\mathbf{i}_\theta\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi}-\frac{\partial(rA_\phi)}{\partial r}\right]+\mathbf{i}_\phi\frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r}-\frac{\partial A_r}{\partial\theta}\right]$$

$$\nabla^2 f=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

Cartesian		*Cylindrical*		*Spherical*
x	=	$\mathrm{r}\cos\phi$	=	$r\sin\theta\cos\phi$
y	=	$\mathrm{r}\sin\phi$	=	$r\sin\theta\sin\phi$
z	=	z	=	$r\cos\theta$
$\mathbf{i}_x$	=	$\cos\phi\,\mathbf{i}_r-\sin\phi\,\mathbf{i}_\phi$	=	$\sin\theta\cos\phi\,\mathbf{i}_r+\cos\theta\cos\phi\,\mathbf{i}_\theta-\sin\phi\,\mathbf{i}_\phi$
$\mathbf{i}_y$	=	$\sin\phi\,\mathbf{i}_r+\cos\phi\,\mathbf{i}_\phi$	=	$\sin\theta\sin\phi\,\mathbf{i}_r+\cos\theta\sin\phi\,\mathbf{i}_\theta+\cos\phi\,\mathbf{i}_\phi$
$\mathbf{i}_z$	=	$\mathbf{i}_z$	=	$\cos\theta\,\mathbf{i}_r-\sin\theta\,\mathbf{i}_\theta$

Cylindrical		*Cartesian*		*Spherical*
r	=	$\sqrt{x^2+y^2}$	=	$r\sin\theta$
ϕ	=	$\tan^{-1} y/x$	=	ϕ
z	=	z	=	$r\cos\theta$
$\mathbf{i}_r$	=	$\cos\phi\,\mathbf{i}_x+\sin\phi\,\mathbf{i}_y$	=	$\sin\theta\,\mathbf{i}_r+\cos\theta\,\mathbf{i}_\theta$
$\mathbf{i}_\phi$	=	$-\sin\phi\,\mathbf{i}_x+\cos\phi\,\mathbf{i}_y$	=	$\mathbf{i}_\phi$
$\mathbf{i}_z$	=	$\mathbf{i}_z$	=	$\cos\theta\,\mathbf{i}_r-\sin\theta\,\mathbf{i}_\theta$

Spherical		*Cartesian*		*Cylindrical*
r	=	$\sqrt{x^2+y^2+z^2}$	=	$\sqrt{\mathrm{r}^2+z^2}$
θ	=	$\cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}}$	=	$\cos^{-1}\frac{z}{\sqrt{\mathrm{r}^2+z^2}}$
ϕ	=	$\cot^{-1} x/y$	=	ϕ
$\mathbf{i}_r$	=	$\sin\theta\cos\phi\,\mathbf{i}_x+\sin\theta\sin\phi\,\mathbf{i}_y+\cos\theta\,\mathbf{i}_z$	=	$\sin\theta\,\mathbf{i}_r+\cos\theta\,\mathbf{i}_z$
$\mathbf{i}_\theta$	=	$\cos\theta\cos\phi\,\mathbf{i}_x+\cos\theta\sin\phi\,\mathbf{i}_y-\sin\theta\,\mathbf{i}_z$	=	$\cos\theta\,\mathbf{i}_r-\sin\theta\,\mathbf{i}_z$
$\mathbf{i}_\phi$	=	$-\sin\phi\,\mathbf{i}_x+\cos\phi\,\mathbf{i}_y$	=	$\mathbf{i}_\phi$

Geometric relations between coordinates and unit vectors for Cartesian, cylindrical, and spherical coordinate systems.