1. a. The expression for the Taylor series of a function $f(x)$ around $x=a$ is:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

i.
$$f(x)|_{x=a=0} = (1+2x)^{1/2}|_{x=0} = 1$$
$$f'(x)|_{x=a=0} = \tfrac{1}{2}(1+2x)^{-1/2}(2)|_{x=0} = 1$$
$$f''(x)|_{x=a=0} = -\tfrac{1}{2}(1+2x)^{-3/2}(2)|_{x=0} = -1$$

$$\Rightarrow f(x) = 1 + \frac{1}{1!}(x-0) + \frac{(-1)}{2!}(x-0)^2 + \ldots$$

so $\boxed{\sqrt{1+2x} \simeq 1+x \text{ for small } x}$

ii.
$$f(x)|_{x=a=0} = (1+2x)^{-1/2}|_{x=0} = 1$$
$$f'(x)|_{x=a=0} = -\tfrac{1}{2}(1+2x)^{-3/2}(2)|_{x=0} = -1$$
$$f''(x)|_{x=a=0} = \tfrac{3}{4}(1+2x)^{-5/2}(2)|_{x=0} = \tfrac{3}{2}$$

$$\Rightarrow f(x) = 1 + \frac{(-1)}{1!}(x-0) + \frac{(3/2)}{2!}(x-0)^2 + \ldots$$

so $\boxed{\sqrt{\frac{1}{1+2x}} \simeq 1-x \text{ for small } x}$

b. $$\sqrt[3]{1+j2} = \boxed{\left\{\begin{matrix} 1.3077e^{j0.1175\pi}, & 1.3077e^{j0.7841\pi}, & 1.3077e^{j1.4508\pi} \\ 1.2196+j0.4718, & -1.0183+j0.8205, & -0.2013-j1.2921 \end{matrix}\right\}}$$

c. $$\mathrm{Re}\left\{e^{j\pi/4}\frac{2+j}{1+j2}\right\} = \boxed{0.9899}$$

2. a. $\boxed{\theta = \pi/2}$ since $\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta = 0 \Rightarrow \cos\theta = 0$ & $\theta = \pi/2$

b. Unit vector $\perp$ $\bar{A}$ & $\bar{B}$: $\hat{n} = \dfrac{\bar{A}\times\bar{B}}{|\bar{A}\times\bar{B}|} = \boxed{-\frac{1}{\sqrt{2}}(\hat{y}+\hat{z})}$

c. $$\nabla\cdot\bar{C} = \frac{\partial C_x}{\partial x} + \frac{\partial C_y}{\partial y} + \frac{\partial C_z}{\partial z} = \boxed{3y-4}$$

d. $$\nabla\times\bar{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3xy & 2z & -4z \end{vmatrix} = \boxed{-2\hat{x} - 3x\hat{z}}$$

e. $\nabla\bar{C}$ not defined; gradient operates on scalars.

$$\nabla^2\bar{C} = \hat{x}\nabla^2 C_x + \hat{y}\nabla^2 C_y + \hat{z}\nabla^2 C_z = \hat{x}\left[\frac{\partial^2 C_x}{\partial x^2} + \frac{\partial^2 C_x}{\partial y^2} + \frac{\partial^2 C_x}{\partial z^2}\right] + \hat{y}\left[\frac{\partial^2 C_y}{\partial x^2} + \frac{\partial^2 C_y}{\partial y^2} + \frac{\partial^2 C_y}{\partial z^2}\right] + \hat{z}\left[\frac{\partial^2 C_z}{\partial x^2} + \frac{\partial^2 C_z}{\partial y^2} + \frac{\partial^2 C_z}{\partial z^2}\right] = \boxed{0}$$

3. General curvilinear system (u, v, w).

a.

Description	h_u	h_v	h_w	du	dv	dw
Cartesian	1	1	1	dx	dy	dz
Cylindrical	1	r	1	dr	$d\phi$	dz
Spherical	1	r	$r\sin\theta$	dr	$d\theta$	$d\phi$

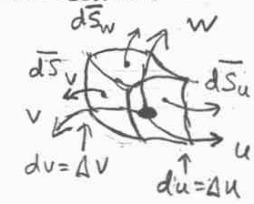
b. Find the gradient of a function $\Phi(u,v,w)$. By definition, $\Delta\Phi = \overline{\nabla\Phi}\cdot\overline{dl}$

$\overline{dl} = h_u du\,\hat{u} + h_v dv\,\hat{v} + h_w dw\,\hat{w}$; $\Delta\Phi = \dfrac{\partial\Phi}{\partial u}du + \dfrac{\partial\Phi}{\partial v}dv + \dfrac{\partial\Phi}{\partial w}dw$

Therefore, $$\boxed{\nabla\Phi = \frac{1}{h_u}\frac{\partial\Phi}{\partial u}\hat{u} + \frac{1}{h_v}\frac{\partial\Phi}{\partial v}\hat{v} + \frac{1}{h_w}\frac{\partial\Phi}{\partial w}\hat{w}}$$

3. c. What are the areas of each surface & the volume of a differential element?

$$\boxed{\begin{aligned}\overline{dS_u} &= h_v h_w\, dv\, dw\, \hat{u} \quad \text{or} \quad dS_u = h_v h_w\, dv\, dw\\ \overline{dS_v} &= h_u h_w\, du\, dw\, \hat{v} \quad \text{or} \quad dS_v = h_u h_w\, du\, dw\\ \overline{dS_w} &= h_u h_v\, du\, dv\, \hat{w} \quad \text{or} \quad dS_w = h_u h_v\, du\, dv\\ &\text{\& } dV = h_u h_v h_w\, du\, dv\, dw\end{aligned}}$$



d. Find divergence & curl of $\bar{A} = A_u \hat{u} + A_v \hat{v} + A_w \hat{w}$

By definition of divergence, if $\Phi = \oiint_S \bar{A} \cdot \overline{dS}$, $\nabla \cdot A \equiv \lim_{\Delta V \to 0} \frac{1}{\Delta V} \Phi = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \oiint_S \bar{A} \cdot \overline{dS}$

Define :

Side #	$\perp$	Location
1′	$-\hat{u}$	$u=0$
1	$\hat{u}$	$u=du$
2′	$-\hat{v}$	$v=0$

Side #	$\perp$	Location
2	$\hat{v}$	$v=dv$
3′	$-\hat{w}$	$w=0$
3	$\hat{w}$	$w=dw$

So

$$\Phi = \iint_1 A_u|_{u+du}\, dS_u - \iint_{1'} A_u|_u\, dS_u' + \iint_2 A_v|_{v+dv}\, dS_v - \iint_{2'} A_v|_v\, dS_v' + \iint_3 A_w|_{w+dw}\, dS_w - \iint_{3'} A_w|_w\, dS_w$$

$$= \left[1 = \frac{\Delta V}{h_u h_v h_w\, du\, dv\, dw}\right] \cdot \Big\{ A_u|_{u+du} (h_v h_w \Delta v \Delta w)_{u+\Delta u} - A_u|_u (h_v h_w \Delta v \Delta w)_u + A_v|_{v+\Delta v} (h_u h_w \Delta u \Delta w)_{v+\Delta v}$$
$$- A_v|_v (h_u h_w \Delta u \Delta w)_v + A_w|_{w+\Delta w} (h_u h_v \Delta u \Delta v)_{w+\Delta w} - A_w|_w (h_u h_v \Delta u \Delta v)_w \Big\}$$

which, by def. of partial derivatives, is the following:

$$\Phi = \frac{\Delta V}{h_u h_v h_w} \left\{ \frac{\partial}{\partial u}(h_v h_w A_u) + \frac{\partial}{\partial v}(h_u h_w A_v) + \frac{\partial}{\partial w}(h_u h_v A_w) \right\}$$

$$\boxed{\text{so } \nabla \cdot \bar{A} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \Phi = \frac{1}{h_u h_v h_w} \left\{ \frac{\partial}{\partial u}(h_v h_w A_u) + \frac{\partial}{\partial v}(h_u h_w A_v) + \frac{\partial}{\partial w}(h_u h_v A_w) \right\}}$$

n^{th} component of curl

By definition of curl, $(\nabla \times A)_n = \lim_{dS_n \to 0} \frac{1}{dS_n} \oint_C \bar{A} \cdot \overline{dl}$

$$\left(\oint_C \bar{A} \cdot \overline{dl}\right)_u = \left\{ \int_v^{v+dv} A_v|_w h_v|_w\, dv + \int_w^{w+dw} A_w|_{v+\Delta v} h_w|_{v+\Delta v}\, dw + \int_{v+\Delta v}^{v} A_v|_{w+\Delta w} h_v|_{w+\Delta w}\, dv + \int_{w+\Delta w}^{w} A_w|_v h_w|_v\, dw \right\}$$

$$\lim_{\Delta v, \Delta w \to 0} \oint_C \bar{A} \cdot \overline{dl} = \left\{ \frac{h_w A_w|_{v+\Delta v} - h_w A_w|_v}{h_v h_w \Delta v} - \frac{h_v A_v|_{w+\Delta w} - h_v A_v|_w}{h_v h_w \Delta w} \right\}$$

$$\Rightarrow \boxed{(\nabla \times \bar{A})_u = \frac{1}{h_v h_w} \left\{ \frac{\partial (h_w A_w)}{\partial v} - \frac{\partial (h_v A_v)}{\partial w} \right\} = u^{th} \text{ comp; etc. for } v^{th} \text{ \& } w^{th}.}$$

e. $\nabla^2 \Phi = \nabla \cdot \nabla \Phi$; Recall $\nabla \Phi = \frac{1}{h_u} \frac{\partial \Phi}{\partial u} \hat{u} + \frac{1}{h_v} \frac{\partial \Phi}{\partial v} \hat{v} + \frac{1}{h_w} \frac{\partial \Phi}{\partial w} \hat{w}$

& $\nabla \cdot \bar{A} = \frac{1}{h_u h_v h_w} \left\{ \frac{\partial}{\partial u} h_v h_w A_u + \frac{\partial}{\partial v} h_u h_w A_v + \frac{\partial}{\partial w} h_u h_v A_w \right\}$

so $$\boxed{\nabla^2 \Phi = \frac{1}{h_u h_v h_w} \left\{ \frac{\partial}{\partial u}\left(\frac{h_v h_w}{h_u} \frac{\partial \Phi}{\partial u} \right) + \frac{\partial}{\partial v}\left(\frac{h_u h_w}{h_v} \frac{\partial \Phi}{\partial v} \right) + \frac{\partial}{\partial w}\left(\frac{h_u h_v}{h_w} \frac{\partial \Phi}{\partial w} \right) \right\}}$$