Review of Taylor Series, Complex Arithmetic, Vector Arithmetic, and Vector Calculus

1. Find the following; parts a-c are unrelated.

a. Show that $\sqrt{1+2x} \approx 1+x$, and find an approximation for $\sqrt{\frac{1}{1+2x}}$.

b. Plot the roots of $\sqrt[3]{1+j2}$ in the complex plane. Express the roots in both rectangular and polar form.

c. What is $\mathrm{Re}\left\{e^{j\frac{\pi}{4}}\frac{(2+j)}{(1+j2)}\right\}$?

2. Find the following.

i. If $\hat{x}$ is a unit vector in the x-direction, compute the following when

$$\vec{A} = 2\hat{x} + \hat{y} - \hat{z} \quad \text{and} \quad \vec{B} = \hat{x} - \hat{y} + \hat{z}.$$

a. What is the angle between the two vectors **A** and **B**?

b. Find a unit vector that is perpendicular to the two vectors **A** and **B**.

ii. If $\vec{C} = 3xy\hat{x} + 2z\hat{y} - 4z\hat{z}$, find the following. (My apologies - the symbol ▼ is intended to be the "del" operator, but my computer is having a problem with its fonts …)

c. What is $\blacktriangledown \cdot \vec{C}$?

d. What is $\blacktriangledown \times \vec{C}$?

e. Would you encounter difficulties in computing $\blacktriangledown \vec{C}$ or $\blacktriangledown^2 \vec{C}$? Compute if possible, and if not, explain the difficulty.

3. A general right-handed curvilinear coordinate system is described by the variables (u, v, w), where $\hat{u} \times \hat{v} = \hat{w}$. Since the incremental coordinate quantities du, dv, and dw do not necessarily have units of length (for example, in polar coordinates, $d\phi$ does not have units of length, but $rd\phi$ does), the differential length elements must be multiplied by coefficients that generally are a function of u, v, and w:

$$d\ell_u = h_u du, \quad d\ell_v = h_v dv, \quad \text{and} \quad d\ell_w = h_w dw.$$

a. What are the h_u, h_v, h_w coefficients for the Cartesian, cylindrical, and spherical coordinate systems?

b. Using the definition of the gradient, find $\nabla\Phi$ (u, v, w) for an arbitrary scalar function Φ.

c. What is the area of each surface and the volume of a differential size volume element in the (u, v, w) space?

d. Using the definitions of curl and divergence, find the curl and divergence of the following arbitrary vector.

$$\bar{A} = A_u\hat{u} + A_v\hat{v} + A_w\hat{w}.$$

e. What is the scalar Laplacian of Φ?

$$\nabla^2\Phi = \nabla(\nabla\Phi)?$$

(Hint: solutions for parts b-e can be checked using known solutions in cylindrical or spherical coordinates.)