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E75 ELECTROMAGNETIC THEORY I

Table 1-2 Geometric relations between coordinates and unit vectors for Cartesian, cylindrical, and spherical coordinate systems*

CARTESIAN		CYLINDRICAL		SPHERICAL
x	=	$r\cos\phi$	=	$r\sin\theta\cos\phi$
y	=	$r\sin\phi$	=	$r\sin\theta\sin\phi$
z	=	z	=	$r\cos\theta$
$\mathbf{i}_x$	=	$\cos\phi\,\mathbf{i}_r-\sin\phi\,\mathbf{i}_\phi$	=	$\sin\theta\cos\phi\,\mathbf{i}_r+\cos\theta\cos\phi\,\mathbf{i}_\theta-\sin\phi\,\mathbf{i}_\phi$
$\mathbf{i}_y$	=	$\sin\phi\,\mathbf{i}_r+\cos\phi\,\mathbf{i}_\phi$	=	$\sin\theta\sin\phi\,\mathbf{i}_r+\cos\theta\sin\phi\,\mathbf{i}_\theta+\cos\phi\,\mathbf{i}_\phi$
$\mathbf{i}_z$	=	$\mathbf{i}_z$	=	$\cos\theta\,\mathbf{i}_r-\sin\theta\,\mathbf{i}_\theta$
CYLINDRICAL		**CARTESIAN**		**SPHERICAL**
r	=	$\sqrt{x^2+y^2}$	=	$r\sin\theta$
ϕ	=	$\tan^{-1}\dfrac{y}{x}$	=	ϕ
z	=	z	=	$r\cos\theta$
$\mathbf{i}_r$	=	$\cos\phi\,\mathbf{i}_x+\sin\phi\,\mathbf{i}_y$	=	$\sin\theta\,\mathbf{i}_r+\cos\theta\,\mathbf{i}_\theta$
$\mathbf{i}_\phi$	=	$-\sin\phi\,\mathbf{i}_x+\cos\phi\,\mathbf{i}_y$	=	$\mathbf{i}_\phi$
$\mathbf{i}_z$	=	$\mathbf{i}_z$	=	$\cos\theta\,\mathbf{i}_r-\sin\theta\,\mathbf{i}_\theta$
SPHERICAL		**CARTESIAN**		**CYLINDRICAL**
r	=	$\sqrt{x^2+y^2+z^2}$	=	$\sqrt{r^2+z^2}$
θ	=	$\cos^{-1}\dfrac{z}{\sqrt{x^2+y^2+z^2}}$	=	$\cos^{-1}\dfrac{z}{\sqrt{r^2+z^2}}$
ϕ	=	$\cot^{-1}\dfrac{x}{y}$	=	ϕ
$\mathbf{i}_r$	=	$\sin\theta\cos\phi\,\mathbf{i}_x+\sin\theta\sin\phi\,\mathbf{i}_y+\cos\theta\,\mathbf{i}_z$	=	$\sin\theta\,\mathbf{i}_r+\cos\theta\,\mathbf{i}_z$
$\mathbf{i}_\theta$	=	$\cos\theta\cos\phi\,\mathbf{i}_x+\cos\theta\sin\phi\,\mathbf{i}_y-\sin\theta\,\mathbf{i}_z$	=	$\cos\theta\,\mathbf{i}_r-\sin\theta\,\mathbf{i}_z$
$\mathbf{i}_\phi$	=	$-\sin\phi\,\mathbf{i}_x+\cos\phi\,\mathbf{i}_y$	=	$\mathbf{i}_\phi$

From Zahn, Electromagnetic Field Theory, 1979

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CARTESIAN COORDINATES

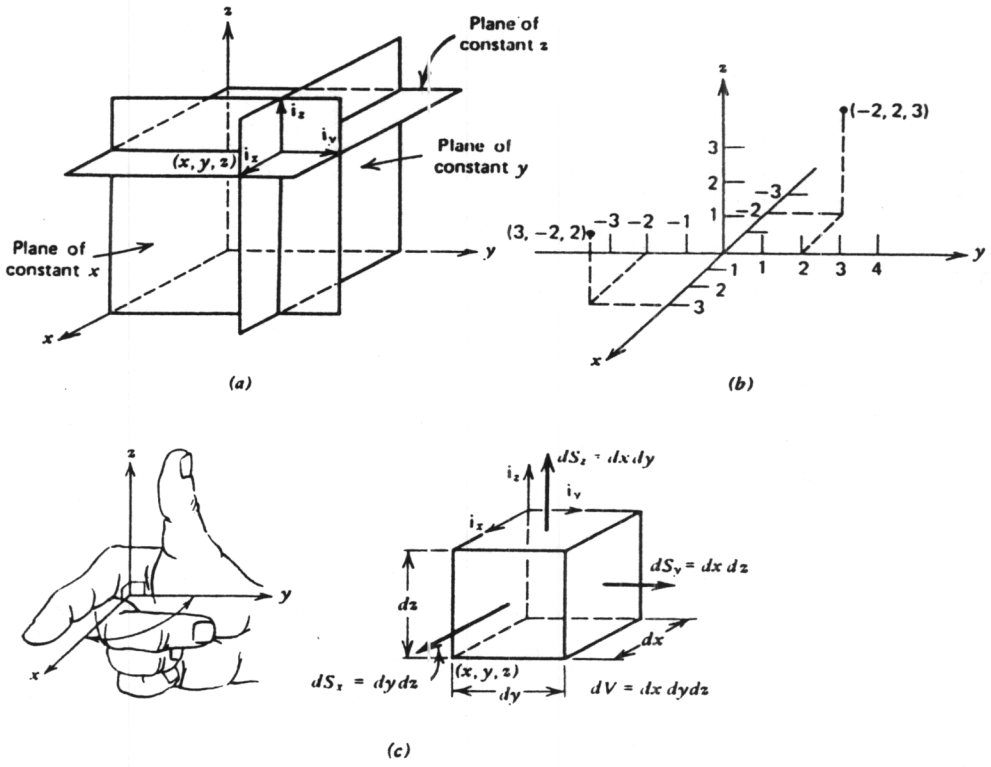

Figure 1-1 Cartesian coordinate system. (*a*) Intersection of three mutually perpendicular planes defines the Cartesian coordinates (*x*, *y*, *z*). (*b*) A point is located in space by specifying its *x*-, *y*- and *z*-directed distances from the origin. (*c*) Differential volume and surface area elements.

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CYLINDRICAL COORDINATES

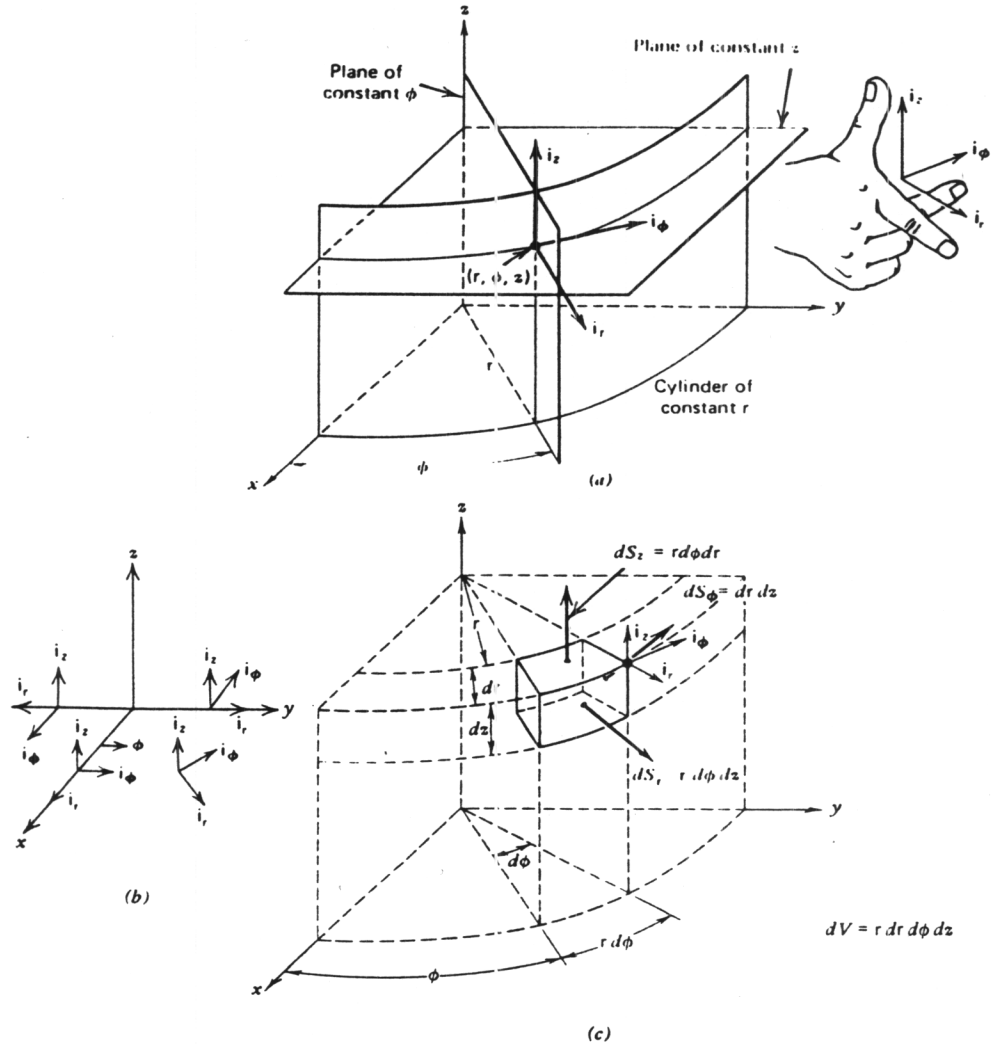

Figure 1-2 Circular cylindrical coordinate system. (*a*) Intersection of planes of constant z and ϕ with a cylinder of constant radius r defines the coordinates (r, ϕ, z). (*b*) The direction of the unit vectors $\mathbf{i}_r$ and $\mathbf{i}_\phi$ vary with the angle ϕ. (*c*) Differential volume and surface area elements.

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SPHERICAL COORDINATES

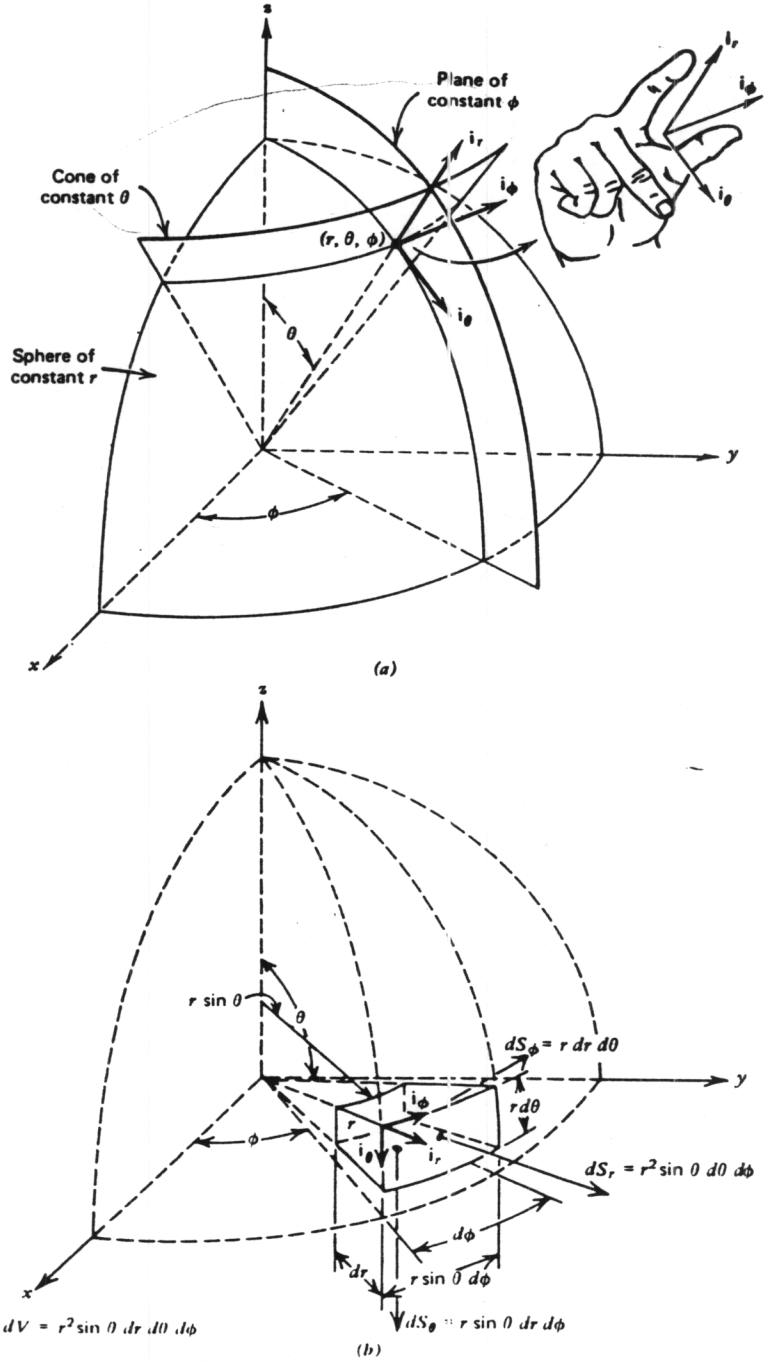
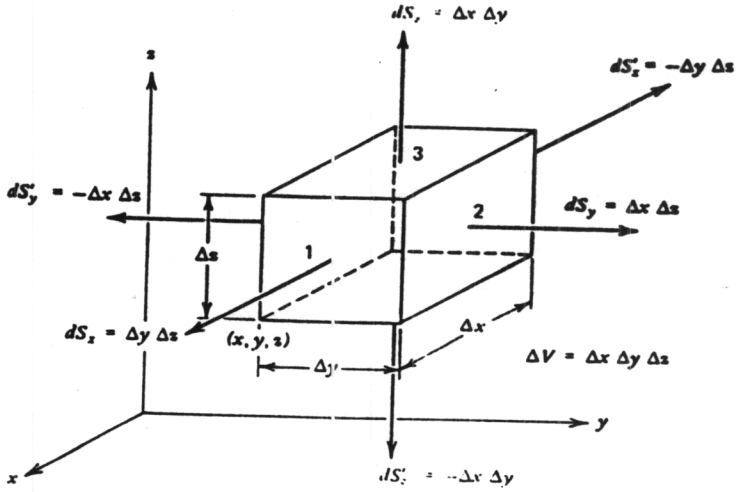
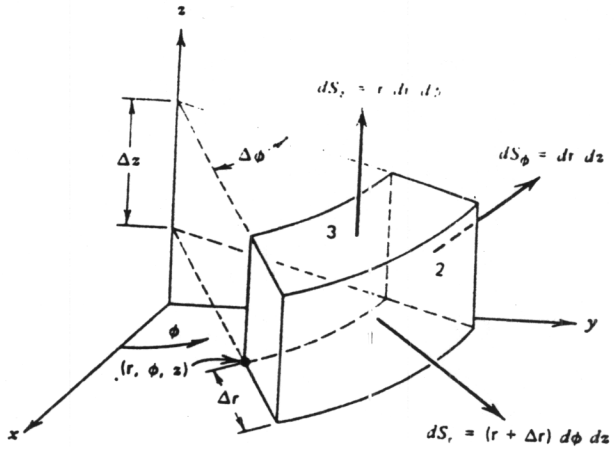
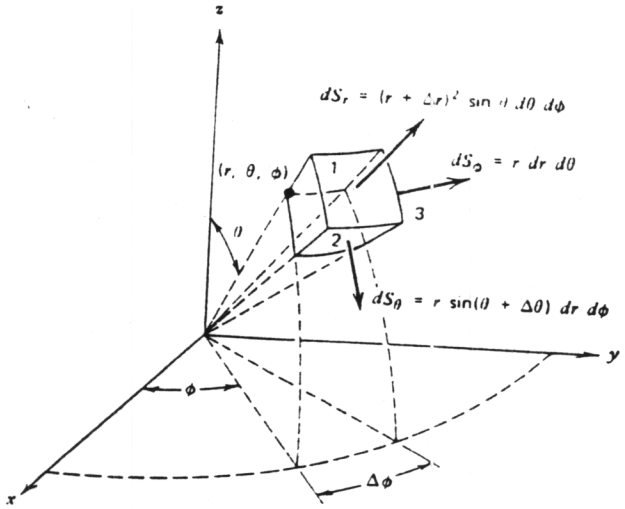


Figure 1-3 Spherical coordinate system. (*a*) Intersection of plane of constant angle ϕ with cone of constant angle θ and sphere of constant radius r defines the coordinates (r, θ, ϕ). (*b*) Differential volume and surface area elements.


$dS_z = \Delta x\,\Delta y$
$dS_x = -\Delta y\,\Delta z$
$dS_y = -\Delta x\,\Delta z$
$dS_y = \Delta x\,\Delta z$
$dS_x = \Delta y\,\Delta z$
$dS_z = -\Delta x\,\Delta y$
$\Delta V = \Delta x\,\Delta y\,\Delta z$
(x, y, z)
z
y
x
1
2
3


$dS_r = r\,d\phi\,dz$
$dS_\phi = dr\,dz$
$dS_z = r\,dr\,d\phi$
$dS_r = (r + \Delta r)\,d\phi\,dz$
Δz
$\Delta\phi$
Δr
(r, ϕ, z)
z
y
x
1
2
3


$dS_r = (r + \Delta r)^2 \sin\theta\,d\theta\,d\phi$
$dS_\theta = r\,dr\,d\phi$
$dS_\theta = r\sin(\theta + \Delta\theta)\,dr\,d\phi$
(r, θ, ϕ)
θ
ϕ
$\Delta\phi$
z
y
x
1
2
3

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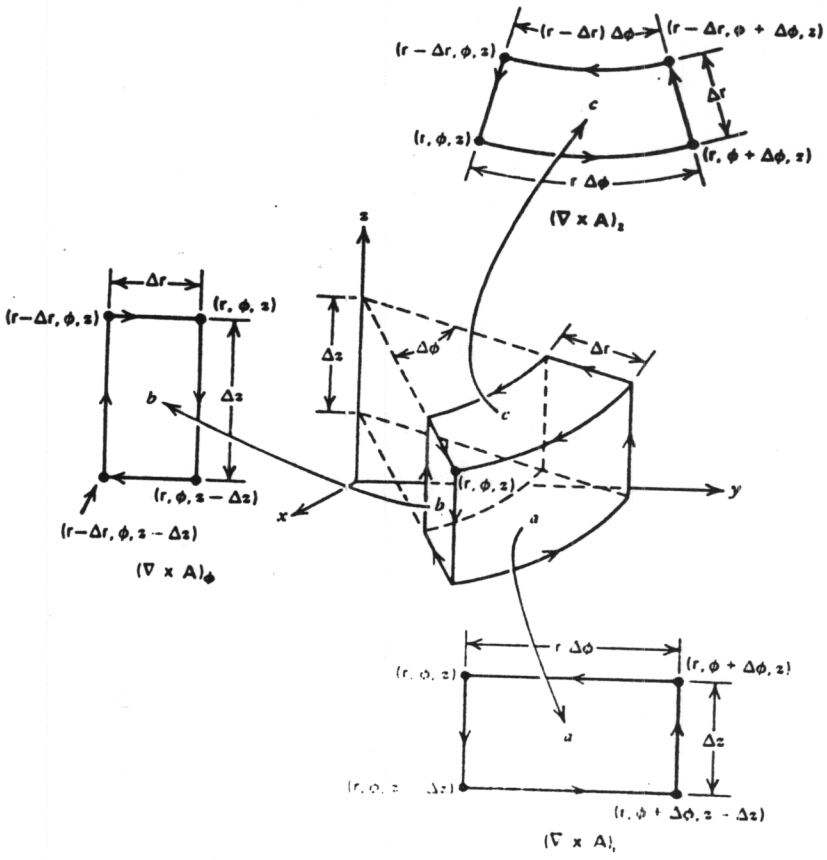
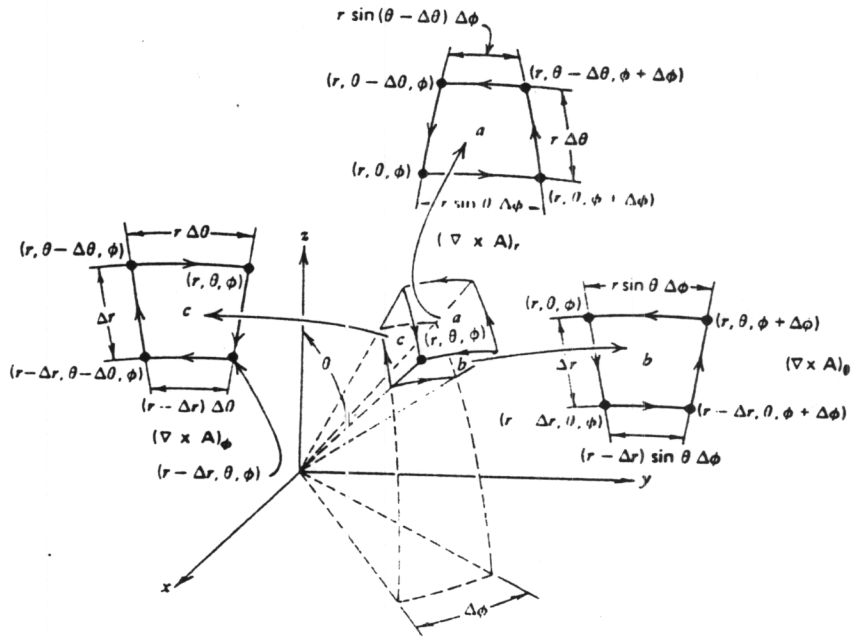

$(r-\Delta r)\,\Delta\phi$
$(r-\Delta r, \phi, z)$
$(r-\Delta r, \phi+\Delta\phi, z)$
Δr
(r, ϕ, z)
$(r, \phi+\Delta\phi, z)$
$r\,\Delta\phi$
$(\nabla \times A)_z$
Δr
$(r-\Delta r, \phi, z)$
(r, ϕ, z)
Δz
Δz
$\Delta\phi$
Δr
$(r, \phi, z-\Delta z)$
$(r-\Delta r, \phi, z-\Delta z)$
$(\nabla \times A)_\phi$
$r\,\Delta\phi$
(r, ϕ, z)
$(r, \phi+\Delta\phi, z)$
Δz
$(r, \phi+\Delta\phi, z-\Delta z)$
$(\nabla \times A)_r$


$r\sin(\theta-\Delta\theta)\,\Delta\phi$
$(r, \theta-\Delta\theta, \phi)$
$(r, \theta-\Delta\theta, \phi+\Delta\phi)$
$r\,\Delta\theta$
(r, θ, ϕ)
$(r, \theta, \phi+\Delta\phi)$
$(\nabla \times A)_r$
$r\,\Delta\theta$
$(r, \theta-\Delta\theta, \phi)$
(r, θ, ϕ)
Δr
$(r-\Delta r, \theta-\Delta\theta, \phi)$
$(r-\Delta r)\,\Delta\theta$
$(\nabla \times A)_\phi$
$(r-\Delta r, \theta, \phi)$
$r\sin\theta\,\Delta\phi$
(r, θ, ϕ)
$(r, \theta, \phi+\Delta\phi)$
Δr
$(r-\Delta r, \theta, \phi)$
$(r-\Delta r, \theta, \phi+\Delta\phi)$
$(r-\Delta r)\sin\theta\,\Delta\phi$
$(\nabla \times A)_\theta$
$\Delta\phi$