E 75 2004

ASSIGNMENTS 7–9 SOLUTIONS

(NOTE : NOT ALL IN ORDER ; CH 5 SOLNS ALSO INCLUDED)

1.10 $\underline{V} = r + jx \Rightarrow V(t) = r\cos\omega t - x\sin\omega t$; $\underline{U} = g + jy \Rightarrow U(t) = g\cos\omega t - y\sin\omega t$

$Re\{\underline{V}\,\underline{U}\,e^{j\omega t}\} = Re\{[(rg - xy) + j(gx + ry)]\,e^{j\omega t}\} = (rg - xy)\cos\omega t - (gx + ry)\sin\omega t$

But $V(t)U(t) = rg\cos^2\omega t + xy\sin^2\omega t - (gx + ry)\sin\omega t\cos\omega t \neq Re\{\underline{V}\,\underline{U}\,e^{j\omega t}\}$

2.18 $A(x,y,z,t) = a(x,y,z)\cos(\omega t + \phi) = a(x,y,z)\cos(-\omega t - \phi)$

$e^{j\omega t}$: $A(x,y,z,t) = Re\{a(x,y,z)e^{j\phi}e^{j\omega t}\} \longleftrightarrow a(x,y,z)e^{j\phi}$

$e^{-i\omega t}$: $A(x,y,z,t) = Re\{a(x,y,z)e^{-i\phi}e^{-i\omega t}\} \longleftrightarrow a(x,y,z)e^{-i\phi}$

2.24 Let $\underline{E} = \bar{E}_R + j\bar{E}_I$ and $\underline{H} = \bar{H}_R + j\bar{H}_I$

$\bar{E} = Re\{\underline{E}\,e^{j\omega t}\} = \bar{E}_R\cos\omega t - \bar{E}_I\sin\omega t$, $\bar{H} = Re\{\underline{H}e^{j\omega t}\} = \bar{H}_R\cos\omega t - \bar{H}_I\sin\omega t$

$\bar{S} = \bar{E}\times\bar{H} = (\bar{E}_R\times\bar{H}_R)\cos^2\omega t + (\bar{E}_I\times\bar{H}_I)\sin^2\omega t - (\bar{E}_R\times\bar{H}_I + \bar{E}_I\times\bar{H}_R)\sin\omega t\cos\omega t$

But $\underline{H}e^{j\omega t} = (\bar{H}_R\cos\omega t - \bar{H}_I\sin\omega t) + j(\bar{H}_R\sin\omega t + \bar{H}_I\cos\omega t)$ and

$Re\{\underline{E}\times\underline{H}e^{j\omega t}\} = \bar{E}_R\times\bar{H}_R\cos\omega t - \bar{E}_R\times\bar{H}_I\sin\omega t - \bar{E}_I\times\bar{H}_R\sin\omega t - \bar{E}_I\times\bar{H}_I\cos\omega t \neq \bar{S}$

3.8 yes; $-z$ direction ; $v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$; $\langle\bar{S}\rangle = \frac{1}{2}Re\{\underline{E}\times\underline{H}^*\} = \frac{1}{2}(E_0\times H_0)\hat{z} = -\frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2\hat{z}$

3.9 No. $\nabla\times\underline{E} = \nabla\times(E_0e^{-jkx}\hat{x}) = 0 \Rightarrow \underline{H} = 0$ (no corresponding H-field exists).

3.10 $\bar{E} = \frac{1}{j\omega\varepsilon_0}\nabla\times\bar{H} = \frac{-1}{j\omega\varepsilon_0}\hat{z}\frac{\partial H_x}{\partial y} = \frac{-40}{\omega\varepsilon_0}e^{j8y}\hat{z}$

$k = \frac{\omega}{v} = 8 \quad \therefore \quad f = \frac{8\times3\times10^8}{2\pi} = 382\ MHz$.

It propagates in $-\hat{y}$ direction.

3.11 (a) $\underline{E} = (j\hat{x} + \hat{y})e^{-jkz} = \hat{x}e^{-j(kz - \frac{\pi}{2})} + \hat{y}e^{-jkz} \Rightarrow \bar{E}(t) = -\hat{x}\sin(\omega t - kz) + \hat{y}\cos(\omega t - kz)$

Right-hand circularly polarized

(b) $\underline{E} = [(1+j)\hat{y} + (1-j)\hat{z}]e^{-jkx} \Rightarrow \bar{E}(t) = \hat{y}\sqrt{2}\cos(\omega t - kx + \frac{\pi}{4}) + \hat{z}\sqrt{2}\cos(\omega t - kx - \frac{\pi}{4})$

Right-hand circularly polarized

(c) $\underline{E} = [(2+j)\hat{x} + (3-j)\hat{z}]e^{-jky} \Rightarrow \bar{E}(t) = \hat{x}\sqrt{5}\cos(\omega t - ky + 0.46) + \hat{z}\sqrt{10}\cos(\omega t - ky - 0.32)$

Left-hand elliptically polarized

(d) $\underline{E} = (j\hat{x} + j2\hat{y})e^{jkz} \Rightarrow \bar{E}(t) = -\hat{x}\sin(\omega t + kz) - \hat{y}\,2\sin(\omega t + kz)$

Linearly polarized.

3.16 $f = f_0$, $v = \frac{1}{\sqrt{\mu_0(4\varepsilon_0)}} = \frac{v_0}{2}$, $\lambda = v/f = \frac{1}{2}(v_0/f_0) = \lambda_0/2$, $k = \frac{2\pi}{\lambda} = 2\left(\frac{2\pi}{\lambda_0}\right) = 2k_0$

3.13 Elliptically polarized wave in general form: $\bar{E}=(\hat{x}\underline{a}+\hat{y}\underline{b})e^{-jkz}$, where $\underline{a}$ and $\underline{b}$ are complex numbers.

Let $\bar{E}=(\hat{x}\underline{a}'+\hat{y}j\underline{a}')e^{-jkz}+(\hat{x}\underline{b}'-\hat{y}j\underline{b}')e^{-jkz}$

$=$ (Left-hand circularly polarized) + (Right-hand circularly polarized)

Then $(\hat{x}\underline{a}+\hat{y}\underline{b})e^{-jkz}=(\hat{x}\underline{a}'+\hat{y}j\underline{a}')e^{-jkz}+(\hat{x}\underline{b}'-\hat{y}j\underline{b}')e^{-jkz}$

$\therefore\ \underline{a}'+\underline{b}'=\underline{a}$ and $j(\underline{a}'-\underline{b}')=\underline{b}\ \Rightarrow\ \underline{a}'=\frac{1}{2}(\underline{a}-j\underline{b})$ and $\underline{b}'=\frac{1}{2}(\underline{a}+j\underline{b})$

Hence, decomposed waves are: $[\hat{x}\frac{1}{2}(\underline{a}-j\underline{b})+\hat{y}\frac{1}{2}(j\underline{a}+\underline{b})]e^{-jkz}$ (left-handed)

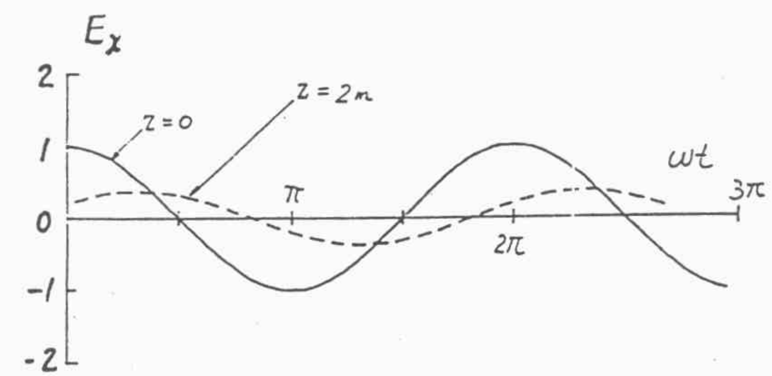
and $[\hat{x}\frac{1}{2}(\underline{a}+j\underline{b})-\hat{y}\frac{1}{2}(j\underline{a}-\underline{b})]$ (right-handed)

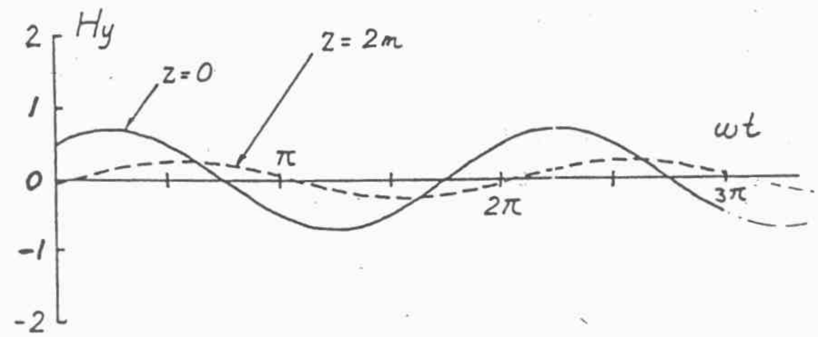
3.23 (a) $E_x=\exp[-j(0.5-j0.5)z]=\exp(-0.5z)\exp(-j0.5z)$

(b) $\bar{H}=\hat{y}\frac{1}{1+j}\exp(-0.5z)\exp(-j0.5z)=\hat{y}(0.5-j0.5)\exp(-0.5z)\exp(-j0.5z)$

(c) $E_x(z,t)=\exp(-0.5z)\cos(\omega t-0.5z)$

(d) $H_y(z,t)=0.707\exp(-0.5z)\cos(\omega t-0.5z-\pi/4)$

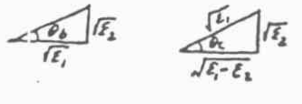




4.3 (i) c (ii) f (iii) b. (iv) a (v) d (vi) e

4.6 impossible; Brewster angle (θ_b) always less than critical angle (θ_c):

$\tan\theta_b=\sqrt{\varepsilon_2/\varepsilon_1}$ $\sin\theta_c=\sqrt{\varepsilon_2/\varepsilon_1}$



$\therefore\ \theta_c>\theta_b$

$\underline{3.21}$ $k = \omega\left[\mu_0 \varepsilon_0\left(\varepsilon_r - j\frac{\sigma}{\omega}\right)\right]^{1/2}$

$\omega = 2\pi \times 10^8$, $\varepsilon_r = 10$, $\sigma = 0.1$

$k = 8.194 - j\,4.818$

$$\left|\frac{E(0.1)}{E(0)}\right| = e^{-0.1 \times 4.818} = 0.618$$

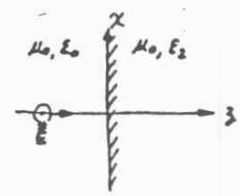
Phase $(x=0.1)$ − Phase $(x=0)$ $= -0.1 \times 8.194 = -0.8194$ rad. $= -46.9^\circ$

$\underline{4.8}$ (a) $\sin\theta_c = \sqrt{1/4} = 1/2 \quad \therefore \theta_c = 30^\circ$

(b) $k_x = k_1 \sin 60^\circ = 2k_0 \frac{\sqrt{3}}{2} = \sqrt{3}\,k_0$, $k_z = k_1 \cos 60^\circ = 2k_0 \frac{1}{2} = k_0$

(c) $k_{tz} = (k_2^2 - k_x^2)^{1/2} = (k_0^2 - 3k_0^2)^{1/2} = -j\sqrt{2}\,k_0$ (d) $z_0 = 1/(\sqrt{2}\,k_0)$

(e) $\underline{R_I} = \dfrac{1+j\sqrt{2}}{1-j\sqrt{2}} = 1\,e^{j109.5^\circ}$



$\underline{4.9}$ (a) $k_1 = \omega\sqrt{\mu_0\varepsilon_0}$, $k_x = 0$, $k_z = k_1$; $k_2 = \omega\sqrt{\mu_0\varepsilon_2}$, $k_{tz} = \sqrt{k_2^2 - k_x^2} = k_2$

Normal impinging: $\bar{E}_1^i = \hat{y}\,E_0\,e^{-jk_z z}$, $\bar{E}_1^r = \hat{y}\,R_I E_0\,e^{jk_z z}$

$R_I = \dfrac{\mu_2 k_z - \mu_1 k_{tz}}{\mu_2 k_z + \mu_1 k_{tz}} = \dfrac{\sqrt{\varepsilon_0} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_0} + \sqrt{\varepsilon_2}}$, $|E_1^{total}| = |E_0|\,|1 + R_I e^{j2k_z z}|$,

Take $|E_0| = 1$; at $z = 0$, $|E_1^{total}| = 0.5 = \left|1 + \frac{\sqrt{\varepsilon_0} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_0} + \sqrt{\varepsilon_2}}\right| \Rightarrow 4\sqrt{\varepsilon_0} = \sqrt{\varepsilon_0} + \sqrt{\varepsilon_2} \quad \therefore \varepsilon_2 = 9\varepsilon_0$

(b) $|E|_{max}$ occurs at $z = -1\,m \Rightarrow e^{-j2k_1}$ must be equal to -1 (since $R_I < 0$)

$\therefore 2k_1 = \pi$ or $\omega = \dfrac{\pi}{2\sqrt{\mu_0\varepsilon_0}} \Rightarrow f = \frac{3}{4} \times 10^8 = 0.75 \times 10^8$ Hz

$\underline{4.10}$ (a) $n_1 \sin 45^\circ = n_2 \sin 30^\circ \Rightarrow \varepsilon_2 = 2\varepsilon_0$

(b) $k_1 = k_0$, $k_x = \frac{1}{\sqrt{2}} k_0$, $k_z = \frac{1}{\sqrt{2}} k_0$; $k_2 = \sqrt{2}\,k_0$, $k_{tz} = \sqrt{k_2^2 - k_x^2} = \sqrt{\frac{3}{2}}\,k_0$

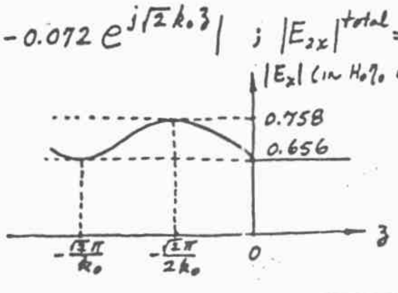
$R_{II} = \dfrac{2-\sqrt{3}}{2+\sqrt{3}} = 0.072$, $T_{II} = 1 + R_{II} = 1.072$

(c) $\underline{\bar{E}}^i = \frac{1}{\sqrt{2}}(\hat{x} - \hat{z})(H_0\eta_0)e^{-j\frac{1}{\sqrt{2}}(k_0 x + k_0 z)}$

$\underline{\bar{E}}^r = \frac{0.072}{\sqrt{2}}(-\hat{x} - \hat{z})(H_0\eta_0)e^{-j\frac{1}{\sqrt{2}}(k_0 x - k_0 z)}$

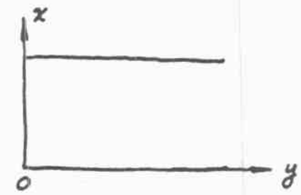
$\underline{\bar{E}}^t = \frac{0.536}{\sqrt{2}}(\sqrt{3}\,\hat{x} - \hat{z})(H_0\eta_0)e^{-j\frac{1}{\sqrt{2}}(k_0 x + \sqrt{3}\,k_0 z)}$

(d) $|E_{1x}|^{total} = \frac{1}{\sqrt{2}}(H_0\eta_0)\left|1 - 0.072\,e^{j\sqrt{2}k_0 z}\right|$; $|E_{2x}|^{total} = \sqrt{\frac{3}{2}}(0.536)(H_0\eta_0) = 0.656\,(H_0\eta_0)$



FYI ADDITIONAL SOLUTIONS FROM CH. 5

CHAPTER 5



5.1 $\nabla\times\bar{H}=j\omega\varepsilon\bar{E}$ and $\nabla\times\bar{E}=-j\omega\mu\bar{H}$ with $\partial/\partial y=0$

$$\Rightarrow \left.\begin{aligned} &-\frac{\partial}{\partial z}E_y=-j\omega\mu H_x\\ &\frac{\partial}{\partial x}E_y=-j\omega\mu H_z\\ &\frac{\partial}{\partial z}H_x-\frac{\partial}{\partial x}H_z=j\omega\varepsilon E_y\end{aligned}\right\}\Rightarrow\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+\omega^2\mu\varepsilon\right)E_y=0$$

Assume $E_y=A(x)e^{-jk_zz}$, then $\frac{d^2A(x)}{dx^2}+\omega^2\mu\varepsilon-k_z^2=0$

$\therefore \frac{d^2A(x)}{dx^2}+k_x^2=0 \Rightarrow A(x)=C_1\cos k_xx+C_2\sin k_xx$, where $k_x^2=\omega^2\mu\varepsilon-k_z^2$

$\therefore E_y=C_1\cos k_xx\,e^{-jk_zz}+C_2\sin k_xx\,e^{-jk_zz}$

$E_y|_{x=0}=0\Rightarrow C_1=0$; $E_y|_{x=a}=0\Rightarrow k_x=\frac{m\pi}{a}$, $m=1,2,3,\cdots$

$\therefore E_y=E_0\sin k_xx\,e^{-jk_zz}$ (denoted C_2 by E_0)

$\bar{H}=\frac{\nabla\times\bar{E}}{-j\omega\mu}=-\hat{x}\frac{k_z}{\omega\mu}E_0\sin k_xx\,e^{-jk_zz}+\hat{z}\,j\frac{E_0}{\omega\mu}k_x\cos k_xx\,e^{-jk_zz}$

$\therefore H_x=-\frac{k_z}{\omega\mu}E_0\sin k_xx\,e^{-jk_zz}$; $H_z=j\frac{k_x}{\omega\mu}E_0\cos k_xx\,e^{-jk_zz}$; and $E_x=E_z=H_y=0$

5.2 Assume $\partial/\partial y=0$ and $H_y=H(x)e^{-jk_zz}$, then $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+\omega^2\mu\varepsilon\right)H_y=\frac{d^2H(x)}{dx^2}+(\omega^2\mu\varepsilon-k_z^2)=0$

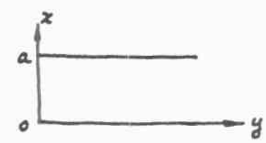
$H(x)=A\cos k_xx+B\sin k_xx$, where $k_x^2=\omega^2\mu\varepsilon-k_z^2$

$\therefore H_y=A\cos k_xx\,e^{-jk_zz}+B\sin k_xx\,e^{-jk_zz}$ and $\bar{E}=\frac{\nabla\times\bar{H}}{j\omega\varepsilon}=\hat{x}\left(\frac{k_z}{\omega\varepsilon}\right)(A\cos k_xx+B\sin k_xx)e^{-jk_zz}$
$+\hat{z}\left(\frac{k_x}{j\omega\varepsilon}\right)(-A\sin k_xx+B\cos k_xx)e^{-jk_zz}$

$E_z|_{x=0}=0\Rightarrow B=0$

$$\therefore\begin{cases}E_x=\frac{k_z}{\omega\varepsilon}H_0\cos k_xx\,e^{-jk_zz} \quad \text{(denoted A by } H_0)\\ E_z=\frac{jk_x}{\omega\varepsilon}H_0\sin k_xx\,e^{-jk_zz}\\ H_y=H_0\cos k_xx\,e^{-jk_zz}\\ H_x=H_z=E_y=0\end{cases}$$

5.3 $f_c=\frac{1}{2a\sqrt{\mu\varepsilon}}=\frac{3\times10^8}{2\times80\times10^3}=1.875$ KHz



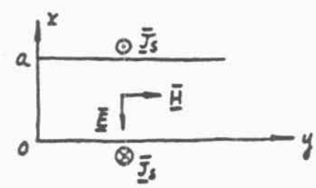
5.4 (a) TE_m: $E_y=E_0\sin\left(\frac{m\pi}{a}x\right)e^{-jk_zz}$

$E_y|_{x=0}=E_y|_{x=a}=0\Rightarrow \rho_s=0$ on both plates

(b) TM_m: $E_x=\frac{k_z}{\omega\varepsilon}H_0\cos\left(\frac{m\pi}{a}x\right)e^{-jk_zz}$ (From Problem 5.2)

at $x=0$, $\rho_s=\hat{x}\cdot\varepsilon\bar{E}|_{x=0}=\frac{k_z}{\omega}H_0e^{-jk_zz}$

at $x=a$, $\rho_s=-\hat{x}\cdot\varepsilon\bar{E}|_{x=a}=-\frac{k_z}{\omega}H_0(-1)^me^{-jk_zz}=(-1)^{m+1}\frac{k_z}{\omega}H_0e^{-jk_zz}$, $m=0,1,2,\cdots$



5.5 TEM in parallel plates $=TM_0$

propagating in $-z$ direction : $\bar{H}=\hat{y}H_0e^{jkz}$, $k=\omega\sqrt{\mu\varepsilon}$

$\bar{E}=\frac{\nabla\times\bar{H}}{j\omega\varepsilon}=\frac{H_0}{j\omega\varepsilon}(-jk\hat{x})e^{jkz}=-\hat{x}H_0\eta e^{jkz}$

at $x=0$, $\bar{J}_s=\hat{x}\times\bar{H}|_{x=0}=\hat{z}H_0e^{jkz}$

at $x=a$, $\bar{J}_s=-\hat{x}\times\bar{H}|_{x=a}=-\hat{z}H_0e^{jkz}$

5.6 $P=\frac{1}{2}|E|^2\frac{wa}{\eta}=\frac{(10^4)^2\times7.1\times10^{-3}\times1.5\times10^{-3}}{2\times120\pi\times(1/\sqrt{2.5})}=2.233$ W

5.7 $|\bar{E}|=2\times10^6$ and $P=\frac{(2\times10^6)^2\times7.1\times10^{-3}\times1.5\times10^{-3}}{2\times120\pi\times(1/\sqrt{2.5})}=89.33$ KW

5.8 From equation (5.16b), with $m=0$,

$\bar{J}_s(y=b)=-\hat{y}\times\bar{H}|_{y=b}=-\hat{z}\left(\frac{k_zE_0}{\omega\mu}\right)\sin\left(\frac{\pi x}{a}\right)e^{-jk_zz}-\hat{x}\left(\frac{jE_0\pi}{\omega\mu a}\right)\cos\left(\frac{\pi x}{a}\right)e^{-jk_zz}$

at $x=\frac{a}{2}$, $\bar{J}_s(y=b)=-\hat{z}\frac{k_zE_0}{\omega\mu}e^{-jk_zz}$, $\hat{z}$ component of current only

$\therefore$ The slot should cut along z at the middle (i.e. at $x=a/2$).