Example 8

A load of 6 Ω is fed by a parallel combination of two batteries. Battery A has an open-circuit voltage of 42 V and an internal resistance of 12 Ω; battery B has an open-circuit voltage of 35 V and an internal resistance of 3 Ω. Determine the current I supplied by battery B, and calculate the power dissipated internally by the battery

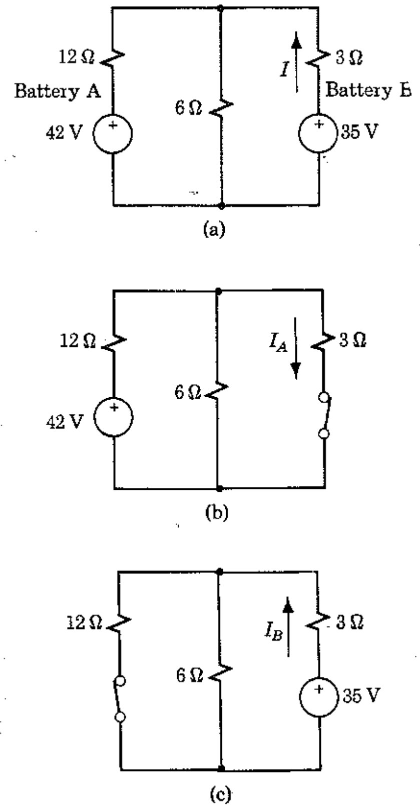


Figure 2.23 Application of the superposition principle

If the batteries are represented by linear models, the circuit is as shown in Fig. 2.23a. With the 35-V source removed (Fig. 2.23b), the component of current due to battery A can be written in one step as

$$I_A = \frac{6}{6+3}\,\frac{42}{\left(12 + \dfrac{3 \times 6}{3+6}\right)} = \frac{2}{3}\left(\frac{42}{12+2}\right) = 2 \text{ A downward}$$

if the principles of current division (Eq. 2-34) and resistance combination (Eqs 2-27 and 2-29) are applied simultaneously

If the 42-V source is removed (the internal resistance of the battery remains), the component of current due to battery B (Fig 2.23c) is

$$I_B = \frac{35}{3 + \dfrac{6 \times 12}{6+12}} = \frac{35}{3+4} = 5 \text{ A upward}$$

Then, by superposition,

$$I = -I_A + I_B = -2 + 5 = 3 \text{ A upward}$$

To calculate the power dissipated in the 3-Ω resistance, we might note that with the 35-V source removed, the power is

$$P_A = I_A^2 R = 2^2 \times 3 = 12 \text{ W}$$

With the 42-V source removed, the power is

$$P_B = I_B^2 R = 5^2 \times 3 = 75 \text{ W}$$

But. the actual power dissipated is *not* 12 + 75 = 87 W. Why not? What principle would be violated in such a calculation?

The answer is that power is not linearly related to voltage or current and therefore superposition cannot be applied in power calculations. Superposition is applicable only to linear effects such as the current response. Having found the current by superposition, we can calculate the power in the 3-Ω resistance as

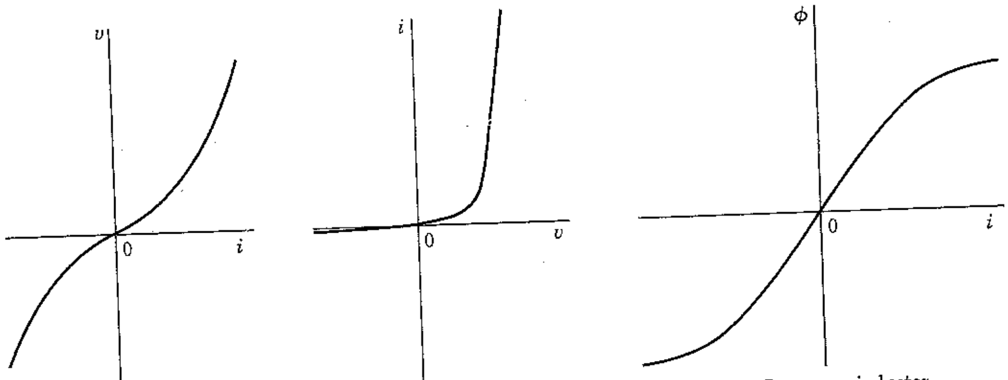
$$P = I^2R = (-I_A + I_B)^2R = 3^2 \times 3 = 27 \text{ W}$$

2-4 NONLINEAR NETWORKS

In the preceding discussions we assumed linearity, but real devices are never strictly linear. In some cases, nonlinearity can be disregarded; linear approximation yields results that predict the behavior of the real devices within acceptable limits. In other cases, nonlinearity is annoying and special steps must be taken to avoid or eliminate its effect; later we shall learn how to use feedback to minimize the distortion introduced in nonlinear electronic amplifiers. Sometimes nonlinearity is desirable or even essential; the distortion that is annoying in an amplifier is necessary in the harmonic generator for obtaining output signals at frequencies that are multiples of the input signal.

NONLINEAR ELEMENTS

A dissipative element for which voltage is not proportional to current is a *nonlinear resistor*. An ordinary incandescent lamp has a characteristic similar to that in Fig. 2.24a; the "resistance" of the filament increases with temperature and, therefore, under steady-state conditions, with current.



(a) Incandescent lamp (b) Semiconductor diode (c) Iron-core inductor

Figure 2.24 Characteristics of nonlinear elements

As we shall see, most electronic devices are inherently nonlinear. Because of its v–i characteristic (Fig. 2.24b), the semiconductor *diode* is useful in discriminating between positive and negative voltages. If a sinusoidal voltage is applied to such a diode, the resulting current has a large dc component and the diode is functioning as a *rectifier*.

To increase the energy-storage capability of an inductor, an iron core with favorable magnetic properties is used. (These properties are discussed in Chapter 20.) In such an inductor, the magnetic flux ϕ is not proportional to current (Fig. 2.24c); at large values of current, a given increment of current produces only a small increment of flux. One interpretation is that "inductance" is not constant. This nonlinearity is troublesome in a power transformer, but it may be useful in a control system.

METHODS OF ANALYSIS

The method of analysis employed depends on the nature of the problem, the form of the data, and the computation aids available. The modern digital computer and analog computer permit the ready solution of problems that were hopelessly time consuming a few years ago, but here we are concerned only with simple series and parallel circuits

ANALYTICAL SOLUTION If an analytical expression for the v–i characteristic can be obtained from physical principles or from experimental data, some simple problems can be solved algebraically. A useful model is the power series

$$i = a_0 + a_1 v + a_2 v^2 + a_3 v^3 + \cdots \tag{2-39}$$

The first two terms provide a linear approximation. The first three terms give satisfactory results in many practical nonlinear problems, although more terms may be necessary in some cases.

Example 9

A voltage $v = V_m \cos \omega t$ is applied to the semiconductor diode of Fig 2.25 Determine the nature of the resulting current.

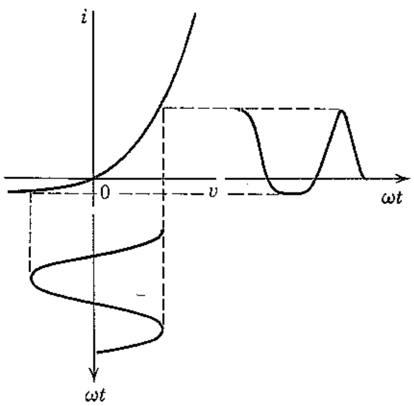


Figure 2.25 Power-series analysis

Assuming that the v–i characteristic can be represented by the first three terms of the power series of Eq. 2-39, the current is

$$i = a_0 + a_1 v + a_2 v^2$$

But we note that for $v = 0$, $i = 0$; therefore, $a_0 = 0$. For the given voltage,

$$i = a_1 V_m \cos \omega t + a_2 V_m^2 \cos^2 \omega t$$

or

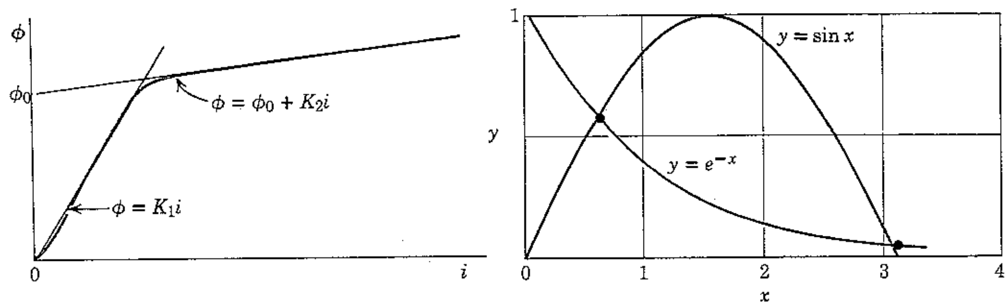
$$i = a_1 V_m \cos \omega t + \frac{a_2 V_m^2}{2}(1 + \cos 2\omega t)$$

$$= \frac{a_2 V_m^2}{2} + a_1 V_m \cos \omega t + \frac{a_2 V_m^2}{2} \cos 2\omega t \tag{2-40}$$

Equation 2-40 indicates that the resulting current consists of a steady or dc component, a "fundamental" component of the same frequency as the applied voltage, and a "second-harmonic" component at the new frequency 2ω.

Depending on the relative magnitudes of a_1 and a_2 and the associated circuitry, the diode of Example 9 could be used as a rectifier, an amplifier, or a harmonic generator. In a specific problem, the coefficients in the power series can be determined by choosing a number of points on the v–i characteristic equal to the number of terms to be included, substituting the coordinates of each point in Eq. 2-39, and solving the resulting equations simultaneously.

PIECEWISE LINEARIZATION Where approximate results are satisfactory, a convenient approach is to represent the actual characteristic by a series of linear "pieces." The characteristic of the iron-core inductor of Fig. 2.26a can be divided into two regions, each approximated by a straight line. Below the "knee" of the curve, the behavior is adequately described by $\phi = K_1 i$; above the knee, a different expression must be used. Usually a trial solution will indicate in which region



(a) Piecewise linearization (b) Graphical solution of equations

Figure 2.26 Analysis based on graphical data

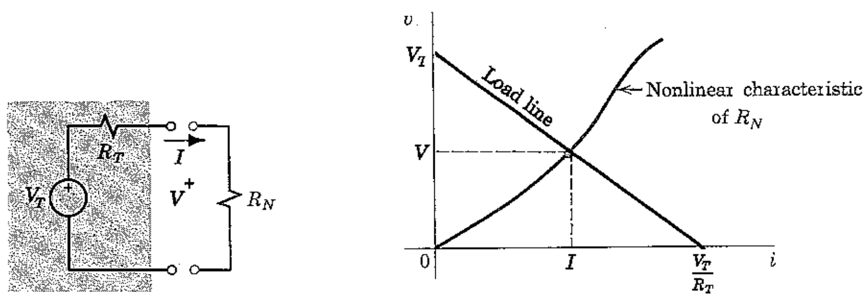
operation is occurring, and the problem is reduced to one in linear analysis. (This approach is used in developing models for electronic devices in Chapter 11.)

GRAPHICAL SOLUTION In piecewise linearization we use graphical data to obtain analytical expressions that hold over limited regions. Sometimes it is desirable to plot given analytical functions to obtain a solution graphically. For example, the equation $e^{-x} - \sin x = 0$ is difficult to solve analytically. If the functions $y = e^{-x}$ and $y = \sin x$ are plotted as in Fig. 2.26b, the simultaneous solutions of these two equations are represented by points whose coordinates satisfy both equations. Therefore, the intersections of the two curves are the solutions to the original equation and can be read directly from the curves. This method is also useful when the characteristic of some portion of the circuit is available in graphical form or in a table of experimental data.

NETWORKS WITH ONE NONLINEAR ELEMENT

If there is a single nonlinear resistance in an otherwise linear resistive network, and if the v–i characteristic of the nonlinear element is known, a relatively simple method of solution is available. This situation occurs frequently in practice and the method of attack deserves emphasis.

The first step is to replace all except the nonlinear element with the Thévenin equivalent shown in Fig. 2.27a as V_T and R_T. The combination of V_T and R_T is



(a) (b)

Figure 2.27 Load-line analysis of a nonlinear network equivalent "insofar as a load is concerned." For any value of load resistance, the terminal voltage is

$$V = V_T - R_T I \tag{2-41}$$

This equation is that of a straight line, the *load line*, with intercepts V_T and V_T/R_T and slope $-R_T$. (For the original network, the intercepts are V_{OC} and I_{SC}.) The graph of the nonlinear characteristic is v as a function of i for the load. The simultaneous satisfaction of these two relations, which yields the values of V and I at the terminals, is the intersection of the two curves (Example 10).

Example 10

The nonlinear element whose characteristic is given in Fig. 2.27b is connected in the circuit of Fig. 2.28a. Determine the current I.

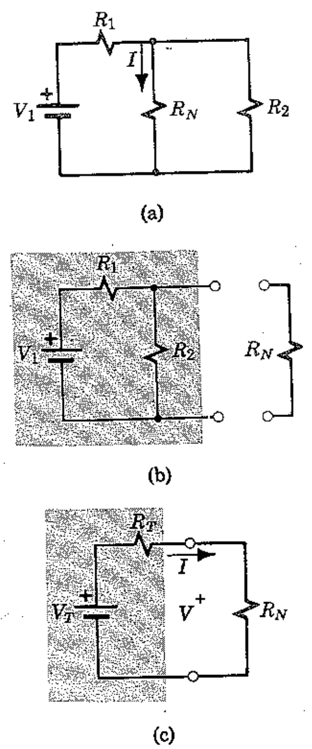


Figure 2.28 Load-line analysis.

First, the nonlinear element is isolated (Fig. 2.28b) and the Thévenin equivalent determined. Considering R_1 and R_2 as a voltage divider,

$$V_T = V_{OC} = \frac{R_2}{R_1 + R_2} V_1$$

The short-circuit current is just $I_{SC} = V_1/R_1$. Therefore, the equivalent resistance is

$$R_T = \frac{V_{OC}}{I_{SC}} = \frac{R_2 V_1}{R_1 + R_2} \frac{R_1}{V_1} = \frac{R_1 R_2}{R_1 + R_2}$$

which is also the resistance seen looking in at the terminals with the voltage source removed.

Next, the load line is drawn with intercepts V_T and V_T/R_T as shown in Fig. 2.27b. The intersection of the load line with the nonlinear characteristic gives the desired current I.

For transistors, motors, and some control system components, the v–i characteristics are represented by a family of curves. The "load" may be the linear element represented by R_T with power supplied by a battery of voltage V_T. Once the load line is drawn, the effect of variations in operating conditions (signals) is clearly visible. Note that if V_T varies during operation, the load line moves parallel to itself.

Exercise 2-12

(a) Determine the parameters G_1, G_2, and I_0 needed to define the characteristic of Fig. E2.12 by piecewise linearization.

(b) If this device is R_N in Fig. 2.27a where $V_1 = 15$ V, $R_1 = 6$ kΩ, and $R_2 = 12$ kΩ, predict I.

(c) After studying the section on networks with one nonlinear element, use the load-line method to predict I in part (b).

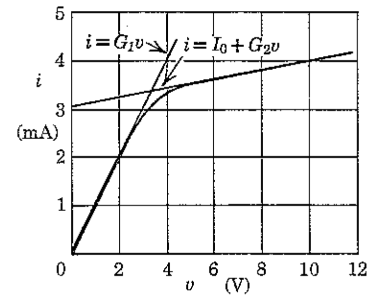


Figure E2.12

Answers: (a) 1 mS, 0.1 mS, 3 mA; (b) 2 mA.

SUMMARY

- The algebraic sum of the currents into a node is zero ($\Sigma i = 0$).
  The algebraic sum of the voltages around a loop is zero ($\Sigma v = 0$).
- The general procedure for formulating equations for circuits is:
  1. Arbitrarily assume a consistent set of currents and voltages.
  2. Write the element equations by applying the element definitions and write the connection equations by applying Kirchhoff's laws.
  3. Combine the element and connection equations to obtain the governing circuit equation in terms of the unknowns.

  The resulting simultaneous equations can be solved by:

  Successive substitution to eliminate all but one unknown, or
  Use of determinants and Cramer's rule.
- Skillful use of loop currents or node voltages may greatly reduce the number of unknowns and simplify the solution.
  Checking is essential because of the many opportunities for mistakes in sign and value.
- Two one-ports are equivalent if they present the same v–i characteristics.
  Two passive one-ports are equivalent if they have the same input resistance.
  For resistances in series, $R_{EQ} = R_1 + R_2 + \cdots + R_n$.
  For conductances in parallel, $G_{EQ} = G_1 + G_2 + \cdots + G_n$.
- For the voltage divider and current divider,

$$V_2 = \frac{R_2}{R_1 + R_2} V \quad \text{and} \quad I_2 = \frac{G_2}{G_1 + G_2} I = \frac{R_1}{R_1 + R_2} I$$

- Insofar as a load is concerned, any one-port network of resistance elements and energy sources can be replaced by a series combination (Thévenin) of an ideal voltage source V_T and a resistance R_T, or by a parallel combination (Norton) of an ideal current source I_N and a conductance G_N, where $V_T = V_{OC}$, $R_T = V_{OC}/I_{SC}$, $I_N = I_{SC}$, and $G_N = I_{SC}/V_{OC} = 1/R_T$.