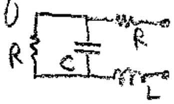
LAM

Engineering 11
Electrical Circuit Analysis
Exam 3 Solutions

2005

① Find $i_L(t)$ if S closes @ $t=0$.

Homogeneous:  $Z(s) = sL + R + \frac{R/sC}{R+1/sC} = \frac{s^2RLC + s(L+R^2C) + 2R}{sRC+1}$

For natural current, use zeros. $R=L=C=1 \Rightarrow s^2+2s+2=0$ or $s=-1\pm j1$

so $i_{L_h} = Ae^{-t}\cos(t+\phi)$

Particular: $\underline{I}_S = I_m\angle 0$; current ÷ $\Rightarrow \underline{I}_L = \frac{\underline{Z}_{RC}}{\underline{Z}_{RC}+\underline{Z}_{RL}}\underline{I}_S$; $\underline{Z}_{RC} = \frac{R/j\omega C}{R+1/j\omega C} = \frac{R}{1+j\omega RC}$; $Z_{RL} = R + j\omega L$

$\Rightarrow \underline{I}_L = \frac{1}{(2-\omega^2 LC) + j\omega(RC + L/R)}\underline{I}_S$

$R=L=C=\omega=1 \Rightarrow \underline{I}_L = \frac{1}{1+j2} I_m\angle 0 = \frac{I_m}{\sqrt5}\angle -\tan^{-1}2$ or $i_{L_p}(t) = \frac{I_m}{\sqrt5}\cos(t-\tan^{-1}(2))$

I.C. $i_L(0^+) = i_L(0^-) = 0$ $\quad i_L(0) = 0 = A\cos\phi + \frac{I_m}{\sqrt5}\cos(-\tan^{-1}2) \Rightarrow \boxed{A = -\frac{I_m}{\sqrt5}\frac{\cos(-\tan^{-1}(2))}{\cos\phi}}$

$\frac{di_L}{dt}\Big|_{0^+} = \frac{v_L(0^+)}{L} = 0$ $\quad \frac{di_L}{dt}(0) = 0 = -A\cos\phi - A\sin\phi - \frac{I_m}{\sqrt5}\sin(-\tan^{-1}(2))$

since $v_C(0^-) = v_C(0^+) = 0 = v_{RL}(0^+)$ & $v_R(0^+) = Ri_L(0^+) = 0$

Combining above eqs: $0 = \frac{I_m}{\sqrt5}\cos(-\tan^{-1}(2)) + \tan\phi\frac{I_m}{\sqrt5}\cos(-\tan^{-1}(2)) - \frac{I_m}{\sqrt5}\sin(-\tan^{-1}(2))$

$0 = 1 + \tan\phi - \tan(-\tan^{-1}(2)) \Rightarrow \tan\phi = -3$ or $\boxed{\phi = -\tan^{-1}(3)}$

so $\boxed{i_L(t) = Ae^{-t}\cos(t+\phi) + \frac{I_m}{\sqrt5}\cos(t-\tan^{-1}(2))u(t) \text{ Amps} \; ; \; A, \phi \text{ given in above boxes}}$

② Find i_1 if $v = 10\cos(100t + \pi/4)$ (s.s. soln.) Use KVL.

$10\angle\pi/4 - 3\underline{I}_1 - j3(\underline{I}_1 - \underline{I}_2) = 0$

$-3\underline{I}_1 - \frac{1}{j0.5}\underline{I}_2 - j3(\underline{I}_2 - \underline{I}_1) = 0$

$\Rightarrow (3+j3)\underline{I}_1 - j3\underline{I}_2 = 10\angle\pi/4$
$(-3+j3)\underline{I}_1 - j\underline{I}_2 = 0$

$\Rightarrow (3+j3)\underline{I}_1 - j3\underline{I}_2 = 10\angle\pi/4$
$(-9+j9)\underline{I}_1 - j3\underline{I}_2 = 0$
$(-6+j12)\underline{I}_1 = 10\angle\pi/4$

$\therefore \underline{I}_1 = \frac{10\angle 45^\circ}{\sqrt{180}\angle 116.6^\circ} \Rightarrow \boxed{i_1(t) = 0.745\cos(100t - 71.6^\circ)\text{ A}}$

③ Note: resistor on left is irrelevant.
$v_0 = v_L$

$\sum_a i = 0 = \frac{1}{L}\int(V_0\delta(t) - v_L)dt - \frac{v_L}{R} - \frac{1}{L}\int v_L dt$

$v_L + \frac{2R}{L}\int v_L dt = \frac{R}{L}V_0\int\delta(t)dt$

Homogeneous: set source $= 0 \Rightarrow Ae^{st}(1 + \frac{2R}{sL}) = 0 \Rightarrow s = -\frac{2R}{L}$

I.C.: $v_L(0^+) + 0 = \frac{R}{L}V_0$ (eval @ $t=0^+$)

$\therefore \boxed{v_0(t) = v_L(t) = \frac{RV_0}{L}e^{-2Rt/L}u(t) \text{ V}}$

④ $Z_{R_LLC} = \frac{R_L(sL + 1/sC)}{R_L + sL + 1/sC} = \frac{R_L(s^2LC+1)}{s^2LC + sR_LC + 1} \Rightarrow H(s) = \frac{Z_{R_LLC}}{R + Z_{R_LLC}} = \frac{R_L(s^2LC+1)}{R(s^2LC + sR_LC + 1) + R_L(s^2LC+1)}$

ⓐ or $\boxed{H(j\omega) = \frac{R_L(1-\omega^2LC)}{(R+R_L)(1-\omega^2LC) + j\omega RR_LC}}$

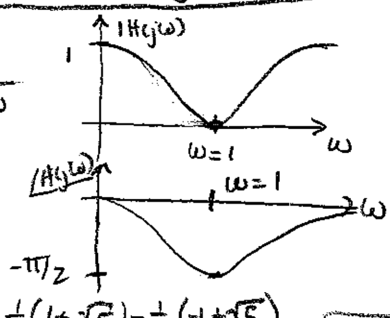
ⓑ Resonant freq $\boxed{\omega_0 = \sqrt{1/LC}}$

ⓒ $H(j0) = \frac{R_L}{R+R_L}$ $\quad H(j\omega_0) = 0$ $\quad H(j\omega)\Big|_{\omega\to\infty} = \frac{R_L}{R+R_L}$

ⓓ let $R=L=C=R_L=1$

$H(j\omega) = \frac{1-\omega^2}{(1-\omega^2) + j\omega}$

$= \frac{1}{1 + j\frac{\omega}{1-\omega^2}}$



ⓔ denom of $H(j\omega) = 1 + j1$ for cutoff freqs.

$\Rightarrow \frac{\omega}{1-\omega^2} = \pm 1 \Rightarrow \omega^2 \pm \omega - 1 = 0$

$\omega_{co} = -\frac12 \pm \sqrt{(\frac14 + 1)} = -\frac12(1 \pm \sqrt5)$ $\quad$ Choose $\omega_{co} > 0$

& $\omega_{co} = \frac12 \pm \sqrt{\frac14 + 1} = \frac12(1 \pm \sqrt5)$

$\Rightarrow \boxed{\omega_{co} = \frac12(-1+\sqrt5), \frac12(+1+\sqrt5)}$

ⓕ $Q = \frac{\omega_{co2} - \omega_{co1}}{\omega_0} = \frac{\frac12(1+\sqrt5) - \frac12(-1+\sqrt5)}{1} = \boxed{1 = Q}$