Exam 3 Review

Ch. 8 2^{nd} order RLC Natural, Particular (dc), Complete

Ch. 9 Sinusoidal Steady State (Particular)

+ Complete (homog + particular)

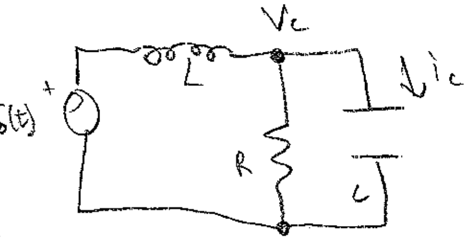
Not → 9.10, 9.11

Ch 10 RMS

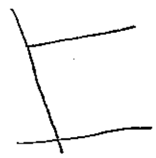
Ch. 14 All

+ Impulse Response

Impulse

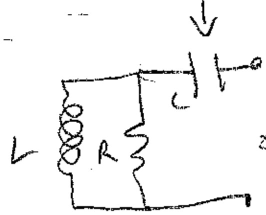


$u(t) = \int \delta(t)\,dt$



$\delta(t) = \dfrac{d[u(t)]}{dt}$

Find i_c



$$Z(s) = \frac{1}{sC} + \frac{sLR}{sL+R}$$

$$= \frac{s^2RLC + sL + R}{sC(sL+R)}$$

Use zeros, $R = L = C = 1$

$$s^2 + s + 1 = 0$$

$$s = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

$$s = -\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - c} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$= -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$$

ω_0 ← resonant

$$= -\alpha \pm j\,\omega_d$$

ω_d ← driving

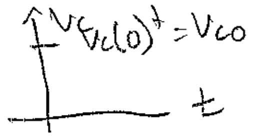
$$i_c = Ae^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t + \phi\right)$$

$$\sum_{V_c} i = 0 = i_L - i_R - i_c$$

$$0 = \frac{1}{L}\int\left[V_0\,\delta(t)\,dt - V_c\right] - \frac{V_c}{R} - C\frac{dV_c}{dt}$$

$$\left.\frac{dV_c}{dt}\right|_{t=0^+} + \frac{1}{RC}\left.V_c\right|_{t=0^+} + \cancel{\frac{1}{LC}\int V_c\,dt}^{\,0} = \frac{1}{LC}\int_{0^-}^{0^+} V_0\,\delta(t)\,dt = \frac{V_0}{LC}u(t)\ \left[\text{but @ } t = 0^+;\ u(t) = 1\right]$$

Want $V_c(0^+) = \begin{cases} V_{co} \\ 0 \end{cases}$



possible

Not possible b/c $\frac{dV_c}{dt}$ would be ∞

$\therefore V_c(0^+)$ must $= 0$ $\therefore \left.\frac{dV_c}{dt}\right|_{t=0^+} = \frac{V_0}{LC}$

$V_c = Be^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t + \theta\right)$ (b/c i_c of same form was found)

$V_c(0) = B\cos\theta = 0 \Rightarrow \theta = \pm\frac{\pi}{2}$

$\left.\frac{dV_c}{dt}\right|_{t=0} = -\frac{B}{2}\cos\theta - \frac{\sqrt{3}}{2}B\sin\theta = V_0$ $(L = C = 1)$

$\boxed{\text{b/c } A\cos\omega t + B\sin\omega t = C\cos(\omega t + \phi) = C\cos\phi\cos\omega t - C\sin\phi\sin\omega t}$

$\theta = -\frac{\pi}{2}$

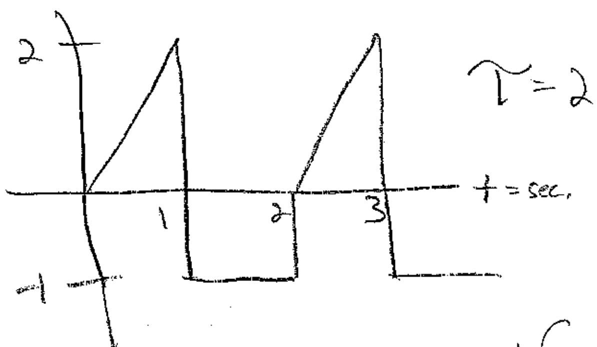
$\left.\frac{dV_c}{dt}\right|_{t=0} = -\frac{\sqrt{3}}{2}B\sin\theta = V_0 \quad B = \frac{2V_0}{\sqrt{3}}$

$V_c(t) = \frac{2V_0}{\sqrt{3}} e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t - \frac{\pi}{2}\right)u(t) \quad t > 0$

$i_c = C\frac{dV_c}{dt} = \left.\frac{2V_0}{\sqrt{3}}e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t - \frac{\pi}{2}\right)\delta(t)\right\} = 0$

$-\frac{\sqrt{3}}{2}\frac{2V_0}{\sqrt{3}}e^{-t/2}\sin\left(\frac{\sqrt{3}}{2}t - \frac{\pi}{2}\right)u(t) - \frac{1}{2}\frac{2V_0}{\sqrt{3}}e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t - \frac{\pi}{2}\right)u(t)$

RMS



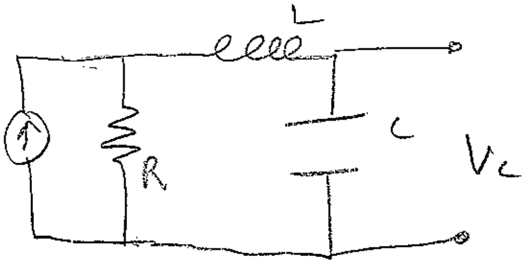
$T = 2$ sec

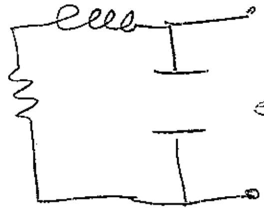
$V_{RMS}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt = \frac{1}{2}\left[\int_0^1 (2t)^2 dt + \int_1^2 (-1)^2 dt\right] = \frac{1}{2}\left[\left(\frac{4}{3}t^3\right)_0^1 + \left[t\right]_1^2\right] = \frac{1}{2}\left(\frac{7}{3}\right) = \frac{7}{6}$

$P_{AVG} = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T \frac{v^2}{R}dt = \frac{V_{RMS}^2}{R}$

Sinusoidal Steady State

(a) $I_m \cos\omega t$
(b) I_m





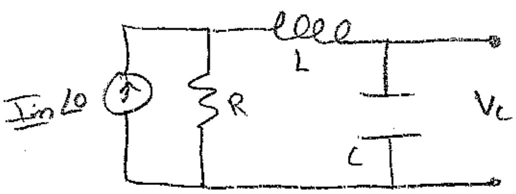
$$Z(s) = \frac{1}{sC} \,//\, (R+sL)$$
$$= \frac{\frac{1}{sC}(R+sL)}{sL+R+1/sC}$$
$$= \frac{sL+R}{s^2LC+sRC+1}$$

$$s^2LC + sRC + 1 = 0$$
$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$
$$s^2 + s + 1 = 0 \qquad s = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$
$$= -\alpha \pm j\omega_d$$

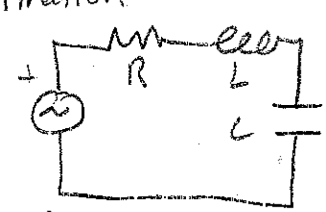
$$V_c = V_{ch} + V_{cp}$$
$$V_{ch} = Ae^{-\alpha t}\cos(\omega_d t + \phi)$$

V_{cp} :

Source transformation + V÷



$\Rightarrow I_mR\angle 0$



$$\underline{V}_c = \frac{Z_c}{R+Z_L+Z_c} I_mR\angle 0 = \left(\frac{-j\left(\frac{1}{\omega C}\right)}{R + j\omega L + \frac{1}{j\omega C}}\right) I_mR\angle 0$$

$R=L=C=\omega=1$

$$= \frac{\frac{1}{\omega C}\angle -\frac{\pi}{2}}{\sqrt{R^2+\left(\omega L - \frac{1}{\omega C}\right)^2}\angle \tan^{-1}}$$

$$= \frac{-j}{1+j-j} I_m\angle 0 = \frac{1\angle -\pi/2}{1} I_m\angle 0 = I_m\angle -\frac{\pi}{2}$$

$$\underline{V}_c = I_m\angle -\frac{\pi}{2}$$

$$V_{cp}(t) = I_m\cos\left(t - \frac{\pi}{2}\right)$$

$$\therefore V_c(t) = Ae^{-\alpha t}\cos(\omega_d t + \phi) + I_m\cos\left(t - \frac{\pi}{2}\right)$$

$$V_c(0^+) = 0 = A\cos\phi + \cancel{I_m\cos\left(-\frac{\pi}{2}\right)} \to 0 \Rightarrow \boxed{\phi = +\frac{\pi}{2}}$$

[b/c $V_c(0^-) = V_c(0^+) = 0$]

$$\left.\frac{dV_c}{dt}\right|_{t=0^+} = \frac{i_c(0^+)}{C} = 0 = -\cancel{\alpha A\cos\phi} \to 0 - \omega_d A\sin\phi - I_m\sin\left(-\frac{\pi}{2}\right)$$

[b/c $i_c(t) = i_L(t)$ and $i_L(0) = 0$]

$$0 = -\omega_d A + I_m \Rightarrow A = \frac{I_m}{\omega_d}$$

$$\boxed{V_c(t) = \left\{\frac{I_m}{\omega_d}e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t - \frac{\pi}{2}\right) + I_m\cos\left(t - \frac{\pi}{2}\right)\right\} u(t) \quad V}$$

b) $V_{cn} = Ae^{-\alpha t}\cos(\omega_d t + \phi)$

$V_{cp} = I_m R$ (b/c capacitor → O.C.)

$V_c = Ae^{-\alpha t}\cos(\omega_d t + \phi) + I_m R$

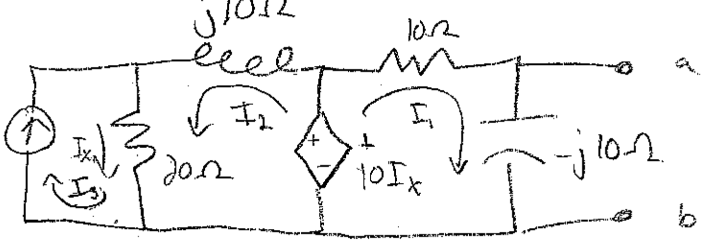
$V_c(0^+) = 0 = A\cos\phi + I_m R$ $\boxed{A = \frac{-I_m R}{\cos\phi}}$

$\left.\frac{dV_c}{dt}\right|_{t=0^+} = \frac{i_c(0^+)}{C} = 0 = -\alpha A\cos\phi - \omega_d A\sin\phi = 0$

then solve

pg. 411

9.11



Find Thev. equivalent @ a, b

Need V_{oc}, I_{sc} Use loops: $I_3 = 2\angle 45°$ (#3)

$V_T = V_{oc}$

$R_T = \frac{V_{oc}}{I_{sc}}$

(#2) $\sum_{I_2} V = 10I_x - (j10\Omega)I_2 - (20\Omega)(I_2 + I_3) = 0$

(#1) $\sum_{I_1} V = 10I_x - 10I_1 + (j10)I_1 = 0$

(#4) $I_x = I_2 + I_3$

(#2) $10\left(I_2 + \underbrace{2\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)}_{2\angle 45°}\right) - j10I_2 - 20\underbrace{(I_2 + I_3)}_{\text{same as 1st term}} = 0$

so $\cancel{-10}\left(I_2 + 2\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)\right) - j\cancel{10}I_2 = 0$

$I_2(\cancel{1+j}) = -2\left(\frac{1}{\sqrt{2}}\right)(\cancel{1+j})$

$I_2 = -\sqrt{2}$

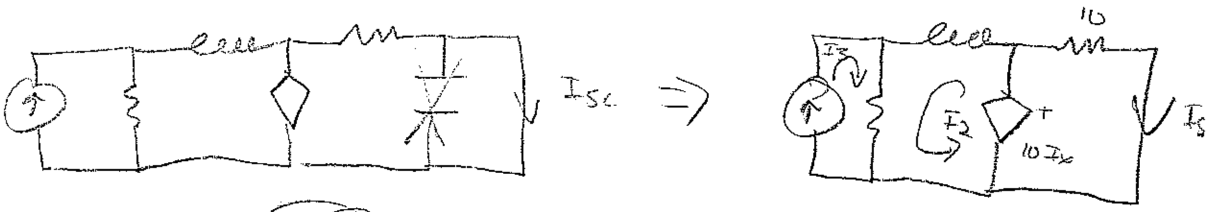
(#4) $I_x = -\sqrt{2} + 2\angle\frac{\pi}{4} = -\sqrt{2} + (\sqrt{2} + j\sqrt{2}) = j\sqrt{2} = \sqrt{2}\angle\frac{\pi}{2} = I_x$

Plug into dependent source + use voltage div.:

$V_T = V_{oc} = \left(\frac{-j}{1-j}\right)\underbrace{(10)\sqrt{2}\angle\frac{\pi}{2}}_{10I_x} = \frac{1\angle\cancel{-\pi/2}}{\cancel{\sqrt{2}}\angle -\pi/4}(10)\cancel{\sqrt{2}}\angle\frac{\pi}{2} = 10\angle\frac{\pi}{4}$

Note: we do not get the book's answer

I_{sc} :



(#3) $I_3 = 2\angle\frac{\pi}{4}$ (#2) $10 I_x - j10 I_2 - 20(I_x) = 0$

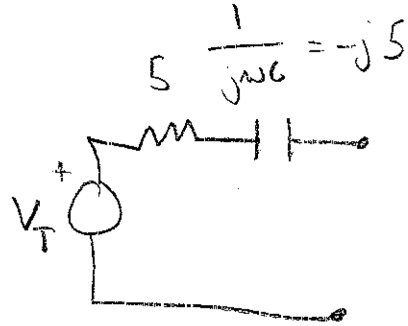
$I_{sc} = \frac{10 I_x}{10} = I_x$ (#1) $I_x = I_2 + I_3 \Rightarrow I_2 = I_x - 2\angle\frac{\pi}{4}$

(#2) $-10 I_x - j10(I_x - 2\angle\frac{\pi}{4}) = 0 \Rightarrow (1+j) I_x = \underbrace{2\angle\frac{\pi}{4}+\frac{\pi}{2}}_{\uparrow \text{ from } j} = 2\angle\frac{3\pi}{4}$

$$I_x = \frac{2\angle\frac{3\pi}{4}}{\sqrt{2}\angle\frac{\pi}{4}} = \sqrt{2}\angle\frac{\pi}{2}$$

$$Z_T = \frac{V_T}{I_{sc}} = \frac{10\angle\frac{\pi}{4}}{\sqrt{2}\angle\frac{\pi}{2}} = \frac{10}{\sqrt{2}}\angle-\frac{\pi}{4}$$

$$Z_T = \frac{10}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) = \boxed{5 - j5}$$



Surprisingly, we do get the book's answer here, even using the V_T we found that is not in agreement with their value.