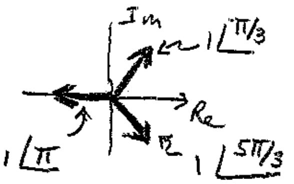
LAM. Engineering 11 2005

Electrical Circuit Analysis
Exam 2 Solutions

1. a $\sqrt[3]{\frac{(1+j)^2}{(1-j)^2}} = \sqrt[3]{\frac{2\angle \pi/2}{2\angle -\pi/2}} = \sqrt[3]{1\angle \pi} = \boxed{1\angle \frac{\pi}{3},\ 1\angle\frac{\pi+2\pi}{3} = 1\angle\pi,\ 1\angle\frac{\pi+4\pi}{3} = 1\angle 5\pi/3}$



b. Find $\underline{C}$ if $\underline{C}^2 - (3-j2)\underline{C} + (1-j3) = 0$

$\underline{C} = \frac{3-j2}{2} \pm \sqrt{\left(\frac{3-j2}{2}\right)^2 - (1-j3)}$

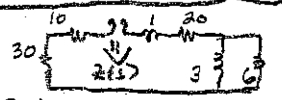
$\boxed{\underline{C} = \frac{3-j2}{2} \pm \sqrt{\frac{3-j6}{2}}}$

OR Let $\underline{C} = a+jb \Rightarrow (a+jb)^2 - (3-j2)(a+jb) + (1-j3) = 0$

$a^2 + 2jab - b^2 - 3a + j2a - j3b - 2b + 1 - j3 = 0$

$\Rightarrow \boxed{a^2 - b^2 - 3a - 2b + 1 = 0\ ;\ 2ab - 3b + 2a - 3 = 0}$ 2 eqs 2 unk

2. Find $i(t)$ if S closes @ $t=0$. For $t<0$, $i = \frac{120}{40} = 3A$, $i_{L=1} = \frac{120}{20} = 6A$.



$i_{L=1}(0^-) = i_{L=1}(0^+) = 6A = -i(0^+)$ For homogeneous soln., $Z(s) = 30 + 10 + s + 20 + \frac{3\cdot 6}{3+6}s = 3s + 60 = 0$
$\Rightarrow s = -20$

$i = i_h + \cancel{i_p}^{\,0} = Ae^{-20t}$; $i(0^+) = -6 = A$ $\therefore \boxed{i(t) = \begin{matrix} 3A & t<0 \\ -6e^{-20t}\ A & t>0 \end{matrix}}$

3. At $t=0$, S: 1 -> 2. Find $v_c(t)$ $t=0^-$: $v_c = \frac{R}{R+R}(2V_s) = V_s$ by voltage ÷

$t>0$: $v_c = v_{ch} + v_{cp}$ $v_{cp} = \frac{R}{R+2R}(-V_s) = -\frac{V_s}{3}$ v_{ch} : $Y(s) = \frac{1}{R} + \frac{1}{2R} + sC = \frac{2sRC + 1 + 2}{2R}$

for v_{ch}, use poles of $Z(s)$ = zeros of $Y(s)$ $\Rightarrow 2sRC + 3 = 0$ or $s = -\frac{3}{2RC}$

$v_c(t) = Ae^{-3t/2RC} + \left(-\frac{V_s}{3}\right)$; $v_c(0) = V_s = A - V_s/3 \Rightarrow A = 4V_s/3$

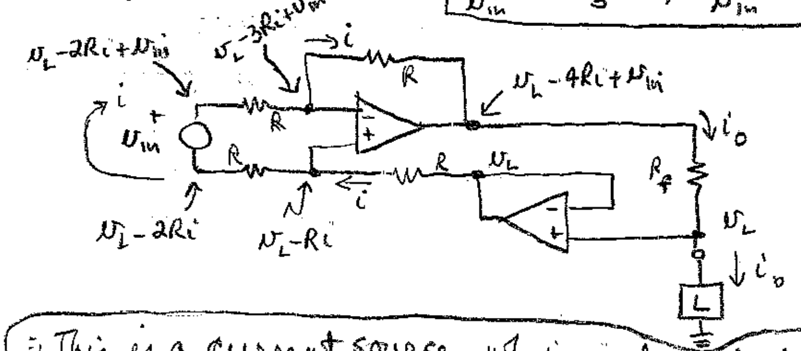
$\therefore \boxed{v_c(t) = \begin{matrix} V_s\ \mathrm{V} & t<0 \\ -\frac{V_s}{3}\left(-4e^{-3t/2RC} + 1\right)\mathrm{V} & t>0 \end{matrix}}$

4. a By voltage ÷, $v_{o1} = \frac{R}{2R}v_{o2} = \frac{v_{o2}}{2}$ (1)

@ inverting input to left op amp, $\sum i = \frac{v_{in} - 0}{R} + \frac{v_{o1} - 0}{4R} + \frac{v_{o2} - 0}{4R} = 0$ (2)

Solving simultaneously, $\boxed{\frac{v_{o1}}{v_{in}} = -\frac{4}{3},\ \frac{v_{o2}}{v_{in}} = -\frac{8}{3}}$

b.



But for left op amp, $v_L - Ri = v_L - 3Ri + v_{in}$

so $i = \frac{v_{in}}{2R}$

$i_o = \frac{(v_L - 4Ri + v_{in}) - v_L}{R_f}$

$= \frac{-4R\left(\frac{v_{in}}{2R}\right) + v_{in}}{R_f} = \boxed{-\frac{v_{in}}{R_f} = i_o}$

∴ This is a current source, w/ i_o independent of the load.