1. In the circuit below, $R_1 = R_2 = L = C = 1$ (with appropriate MKS units). The capacitor has an initial voltage of $V_0 = 1$ V, and the inductor has an initial current of $I_0 = 1$ A. (25 points)

a. Fill in the table below for the initial voltages and currents at $t = 0^+$.

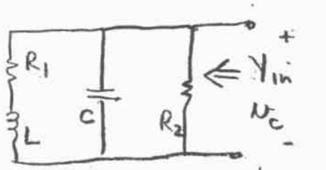
$t = 0^+$	v	i
R_1	1 V	1 A
R_2	1 V	1 A
L	-2 V	1 A
C	1 V	0 A

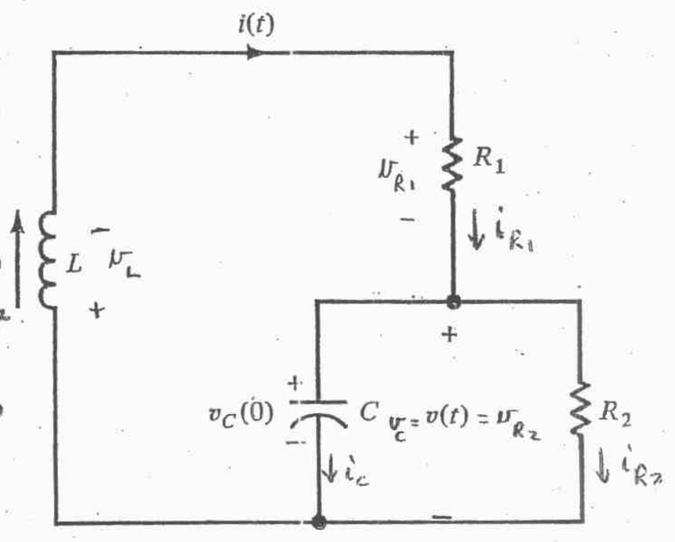
What are the initial values of $\frac{dv_C}{dt}$ and $\frac{di_L}{dt}$ at $t = 0^+$?

$\left.\frac{dv_c}{dt}\right|_{t=0^+} = \frac{i_c(0^+)}{C} = 0$ V/s

$\left.\frac{di_L}{dt}\right|_{t=0^+} = \frac{v_L(0^+)}{L} = -2$ A/s

b. Find the voltage $v_C(t)$ across the capacitor (natural response) for $t \geq 0$.





$Y_{in} = sC + \frac{1}{R_2} + \frac{1}{sL+R_1}$

$= \frac{s^2R_2LC + sL + sR_1R_2C + R_1 + R_2}{R_2(sL+R_1)}$

zeros for v_{cn} : $s^2 + 2s + 2 = 0$

$s = -1 \pm j$

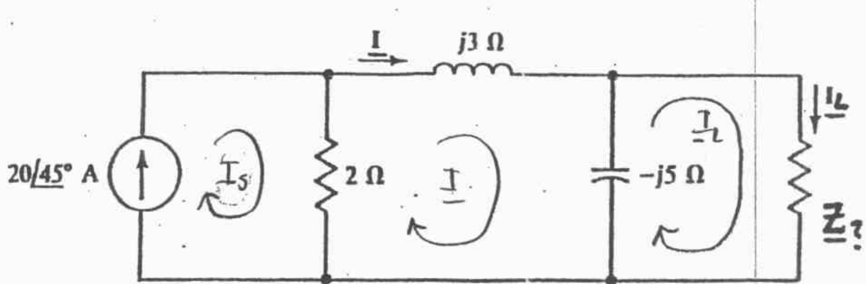
$v_{cn} = Ae^{-t}\cos(t+\phi)$ V $t \geq 0$

$v_{cn}(0) = 1 = A\cos\phi$ using $\Rightarrow A = \frac{1}{\cos\phi} = \frac{1}{\cos(-\pi/4)} = \sqrt{2}$

$\left.\frac{dv_{cn}}{dt}\right|_{t=0} = 0 = -A\cos\phi - A\sin\phi \Rightarrow \tan\phi = -1$ or $\phi = -\pi/4$

$v_{cn}(t) = \sqrt{2}\, e^{-t}\cos(t - \pi/4)\, u(t)$ V

2. In the circuit below, find $\underline{Z}_?$ and $\underline{I}$ if $\underline{I}_L = 6.85 \angle -7°$. Is Z a pure resistance, pure inductance, pure capacitance, or combination (specify R and L or R and C)? If Z is an L or a C, please indicate what additional information you would need to know to specify its value. If Z is an R, please provide its value. (25 points)



$\underline{I}_S = 20 \angle 45°$ $\quad \underline{I}_L = 6.85 \angle -7°$

Middle Loop: KVL $\Sigma v = 0 = j3\,\underline{I} - j5(\underline{I} - \underline{I}_L) + 2(\underline{I} - \underline{I}_S)$

$(j3 - j5 + 2)\,\underline{I} + j5(6.85 \angle -7°) - 40 \angle 45° = 0$

$$\underline{I} = \frac{-4.174 - j34 + 28.28 + j28.28}{2 - j2} = 8.76 \angle 31.65°$$

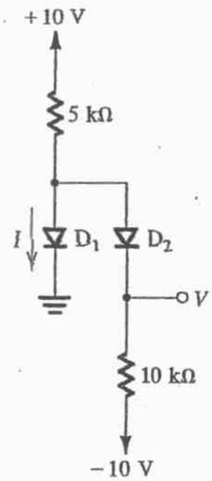
By current divider, $\frac{\underline{I}_L}{\underline{I}} = \frac{-j5}{Z - j5} = \frac{6.85 \angle -7°}{8.76 \angle 31.65°} = .782 \angle -38.65°$

$$Z - j5 = \frac{5 \angle -90°}{.782 \angle -38.65°} = 6.39 \angle -51.4°$$

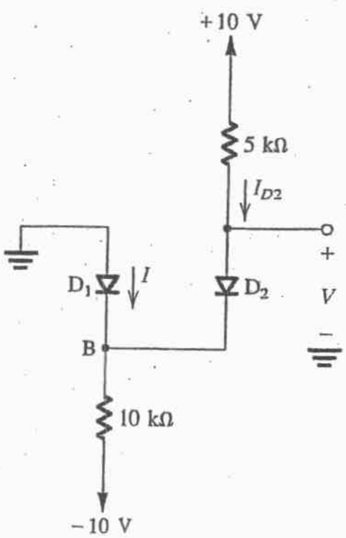
$$\boxed{Z = j5 + 4 - j5 = 4\,\Omega}$$

3. Assume all diodes are ideal. In the circuits below, find I and V as indicated. (25 points)

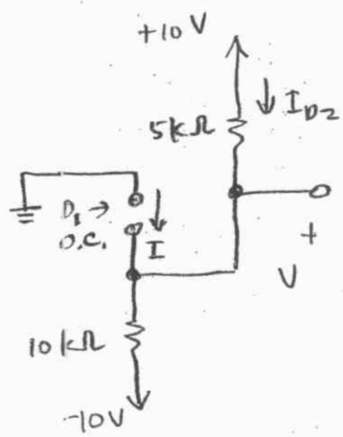
a.



b.



Assume D_1 off; D_2 on. Find i, v & check assumptions.



$I = I_{D1}$

$0 - V = V_{D1}$

$I_{D2} = \frac{10-(-10)}{(10+5)k} = \frac{4}{3} mA$; $V_{D2} = 0$ since S.C.

$V = -10 + 10k(I_{D2}) = -10 + 13.3 = 3.3 V$

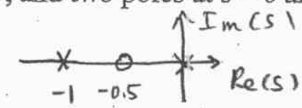
$V_{D1} = 0 - V = 0 - 3.3 = -3.3 V$ } consistent w/ D_1 off ✓

$I_{D1} = I = 0$ since O.C.

$I_{D2} = \frac{4}{3} mA > 0$, $V_{D2} = 0$ } consistent w/ D_2 on ✓

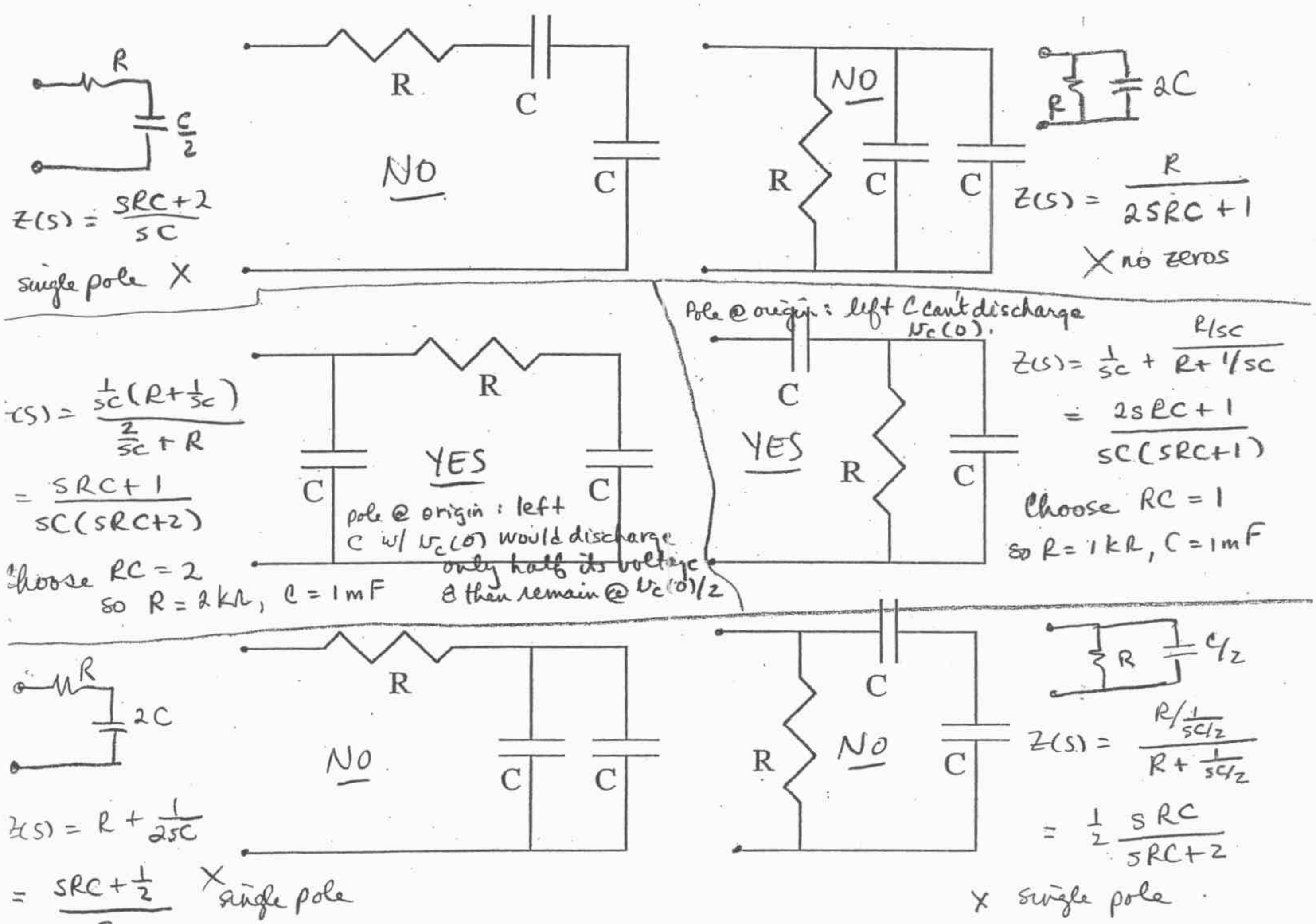
4. A circuit has one zero at s = (−)0.5, and two poles at s = 0 and s = -1. (Impedance) (25 points)

a. Plot the pole-zero diagram.



b. Write the *forms* of the natural current and natural voltage that might occur in this circuit in terms of unknown constant coefficients. $i_n = Ae^{-0.5t}$ $v_n = B_1 + B_2 e^{-t}$

c. If you knew that the circuit contained two identical capacitors and one resistor, choose the configuration from those shown below that would have the above impedance function. Specify the values of R and C. What is the physical meaning of the pole at the origin (i.e. which element is responsible for it, and what IC could cause it to be part of the solution)?



$Z(s) = \frac{sRC+2}{sC}$ single pole X

$Z(s) = \frac{R}{2sRC+1}$ X no zeros

$Z(s) = \frac{\frac{1}{sC}(R+\frac{1}{sC})}{\frac{2}{sC}+R} = \frac{sRC+1}{sC(sRC+2)}$

Choose RC = 2 so R = 2kΩ, C = 1mF

pole @ origin : left C w/ $v_C(0)$ would discharge only half its voltage & then remain @ $v_C(0)/2$

Pole @ origin : left C can't discharge $v_C(0)$.

$Z(s) = \frac{1}{sC} + \frac{R/sC}{R+1/sC} = \frac{2sRC+1}{sC(sRC+1)}$

Choose RC = 1 so R = 1kΩ, C = 1mF

$Z(s) = R + \frac{1}{2sC} = \frac{sRC+\frac{1}{2}}{sC}$ X single pole

$Z(s) = \frac{R/\frac{1}{sC/2}}{R+\frac{1}{sC/2}} = \frac{1}{2}\frac{sRC}{sRC+2}$ X single pole