LAM

ENGINEERING 11
PHYSICAL SYSTEMS ANALYSIS I

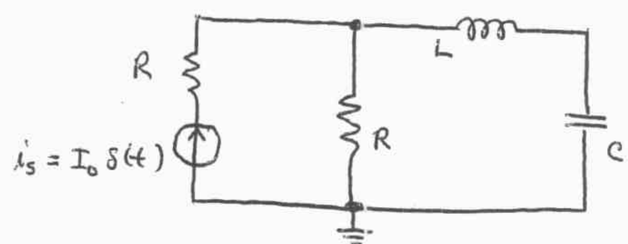
December 4, 2002

EXAM 3, part 2
CLOSED BOOK, CLOSED NOTES, CLOSED MOUTHS

Due at 5PM on Friday, December 6, 2002

1. Find the impulse response of the circuit below. Let R = L = C = 1 (with appropriate units). The source is a current $i_s(t) = I_0\delta(t)$. Find $v_c(t)$ for $t > 0$. Please also explain which other element(s) will experience the impulse in current passing through it/them.

$i_L(0^-) = v_C(0^-) = 0$



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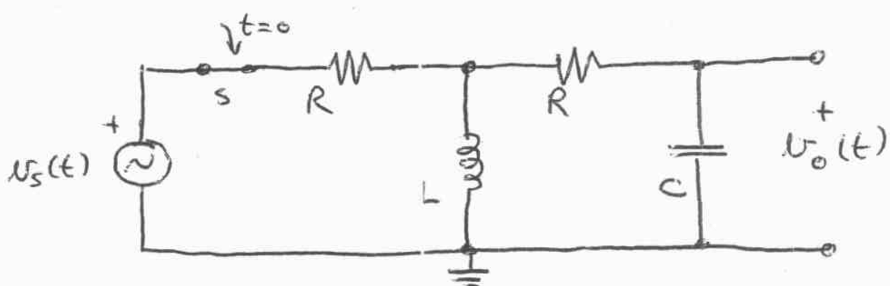
2. Find the complete response of the circuit shown below. A switch closes at time t = 0 to connect the source $v_s(t) = V_0\sqrt{2} * \cos(t)$. Let R = L = C = 1 (with appropriate units).

$i_L(0^-) = v_C(0^-) = 0$



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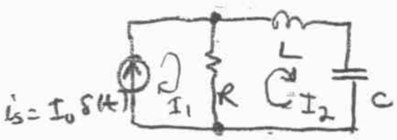
R in series w/ source is irrelevant.



$$-R(I_2 - I_0\delta(t)) - L\frac{dI_2}{dt} - \frac{1}{C}\int I_2\,dt = 0$$

$$\frac{dI_2}{dt} + \frac{R}{L}I_2 + \frac{1}{LC}\int I_2\,dt = \frac{RI_0}{L}\delta(t)$$

let $I_2 = \frac{dq_2}{dt}$

$$\frac{d^2q_2}{dt^2} + \frac{R}{L}\frac{dq_2}{dt} + \frac{1}{LC}q_2 = \frac{RI_0}{L}\delta(t) \quad (*)$$

Natural: $s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + s + 1 = 0 \Rightarrow s = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 1} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

IC: $\frac{dq_2}{dt}\Big|_{t=0^+} + \frac{R}{L}q_2(0^+) = \frac{RI_0}{L} \Rightarrow q_2(0^+) = 0$

$(\int * \, dt)$ $\qquad \frac{dq_2}{dt}\Big|_{t=0^+} = \frac{RI_0}{L}$

$$v_C = \frac{q_2}{C} = Ae^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t + \phi\right)$$

$v_C(0^+) = q_2(0^+) = 0 = A\cos\phi \Rightarrow \phi = -\pi/2$

$\frac{dv_C}{dt}\Big|_{t=0^+} = \frac{dq_2}{dt}\Big|_{t=0^+} = \frac{RI_0}{L} = I_0 = -\frac{A}{2}\cancel{\cos\phi} - \frac{A\sqrt{3}}{2}\sin\phi \Rightarrow A = \frac{2I_0}{\sqrt{3}}$

$$\boxed{v_C(t) = \frac{2I_0}{\sqrt{3}}e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t - \pi/2\right)u(t) \text{ V}}$$

EXAM 3, part 2

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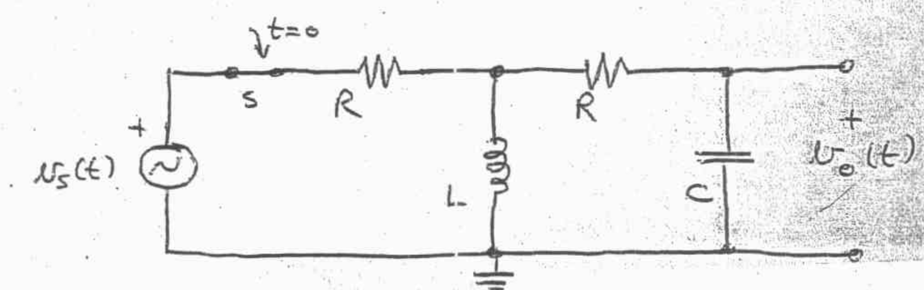
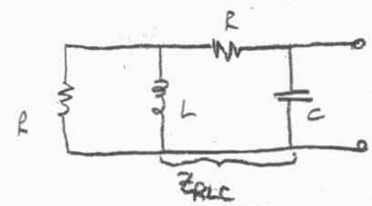
$i_L(0^-) = v_C(0^-) = 0$





$$Z(s) = \frac{\left[\left(\frac{sLR}{sL+R}\right)+R\right]/sC}{\frac{sLR}{sL+R} + R + \frac{1}{sC}} = \frac{2s+1}{2s^2+2s+1}$$

$$s^2+s+\tfrac{1}{2} = 0 \Rightarrow s = -\tfrac{1}{2} \pm \sqrt{\tfrac{1}{4}-\tfrac{1}{2}} = -\tfrac{1}{2} \pm j\tfrac{1}{2}$$

$$v_{cn} = Ae^{-t/2}\cos\left(\frac{t}{2}+\phi\right)$$

Forced:
$$\underline{V}_C = \frac{Z_c}{R+Z_c}\underline{V}_L = \frac{Z_c}{R+Z_c}\,\frac{Z_{RLC}}{R+Z_{RLC}}\underline{V}_s \qquad Z_{RLC} = \frac{sL\left(R+\frac{1}{sC}\right)}{sL+R+\frac{1}{sC}} = \frac{s(s+1)}{s^2+s+1}$$

$$= \frac{1/sC}{R+1/sC}\;\frac{\frac{s(s+1)}{s^2+s+1}}{1+\frac{s(s+1)}{s^2+s+1}}\underline{V}_s = \frac{1}{\cancel{s+1}}\;\frac{s(\cancel{s+1})}{s^2+s+1+s(s+1)} = \frac{s}{2s^2+2s+1}\underline{V}_s$$

Let $s \to j\omega$ $(\omega = 1)$
$$\frac{\underline{V}_c}{\underline{V}_s} = \frac{j}{1-2+j2} = \frac{1\angle 90}{\sqrt{5}\angle 116.56} = 0.447\angle{-26.56^\circ}$$

$$v_{cf} = 0.447\sqrt{2}\cos(t-26.56^\circ)$$

$$v_c = v_{cn} + v_{cf} = Ae^{-t/2}\cos\left(\frac{t}{2}+\phi\right) + 0.447\sqrt{2}\cos(t-26.56^\circ)$$

$$v_c(0^+) = 0 \Rightarrow A\cos\phi + 0.447\sqrt{2}\cos(-26.56^\circ) = 0$$

$$\left.\frac{dv_c}{dt}\right|_{t=0^+} = \frac{i_c(0^+)}{C} = \frac{V_0\sqrt{2}}{2RC} = \frac{V_0\sqrt{2}}{2} = -\tfrac{1}{2}A\cos\phi - \frac{A}{2}\sin\phi - .447\sqrt{2}\sin(-26$$

2 eq, 2 unk. $\Rightarrow$ solve.