1. Find the maximum and minimum values of the function $f(x,y) = 4x+6y$ on the *disk* $x^2+y^2 \leq 1$.

Find interior critical pts., then check the boundary

<u>crit pts</u>: set $\nabla f = \vec{0}$. $\nabla f = (4,6)$: never $\vec{0}$, so no crit. pts.

<u>boundary</u> Use Lagrange multipliers. The constraint is $g(x,y) = x^2+y^2=1$.

$\nabla g = (2x, 2y)$, which is $\vec{0}$ only at $(0,0)$, which is not on the curve $g=1$. So the max & min occur at a pt.

satisfying $\nabla f = \lambda \nabla g$

$$(4,6) = \lambda(2x, 2y)$$

Get 3 eqns:

$$\left.\begin{aligned} 4 &= 2\lambda x \\ 6 &= 2\lambda y \\ x^2+y^2 &= 1 \end{aligned}\right\} \begin{aligned} &\longrightarrow \lambda = \frac{2}{x} \\ &\longrightarrow \lambda = \frac{3}{y} \end{aligned}$$

So $\frac{2}{x} = \frac{3}{y}$, or $2y = 3x$, or $y = 3/2\, x$

Then $x^2 + \left(\frac{3}{2}x\right)^2 = 1$

$x^2 + \frac{9}{4}x^2 = 1$

$\frac{13}{4}x^2 = 1$

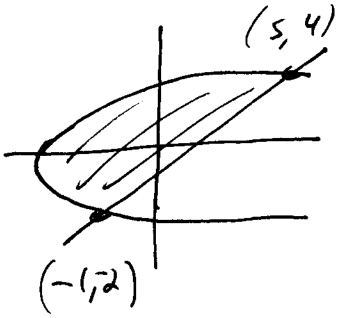
$x^2 = \frac{4}{13}$, $x = \pm \frac{2}{\sqrt{13}}$

So the possible extrema are $\left(\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right)$ & $\left(\frac{-2}{\sqrt{13}}, \frac{-3}{\sqrt{13}}\right)$

max value: $4 \cdot \frac{2}{\sqrt{13}} + 6 \cdot \frac{3}{\sqrt{13}} = \frac{26}{\sqrt{13}}$

min value: $4\left(\frac{-2}{\sqrt{13}}\right) + 6\left(\frac{-3}{\sqrt{13}}\right) = \frac{-26}{\sqrt{13}}$

2. Evaluate $\iint_R y\, dA$, where R is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.



$y^2 = 2x+6$, or $x = \frac{1}{2}y^2 - 3$

$y = x-1$, or $x = y+1$

At pts. of intersection, $\frac{1}{2}y^2 - 3 = y+1$

$$y^2 - 6 = 2y + 2$$

$$y^2 - 2y - 8 = 0$$

$$y = 4, -2$$

So pts. of intersection are $(-2,-1)$ & $(5,4)$

Easiest as a Type II region: $\frac{1}{2}y^2 - 3 \le x \le y+1$

$$-2 \le y \le 4$$

$$\iint_R y\, dA = \int_{-2}^{4} \int_{\frac{1}{2}y^2-3}^{y+1} y \, dx\, dy$$

$$= \int_{-2}^{4} \left(xy \Big|_{\frac{1}{2}y^2-3}^{y+1} \right) dy = \int_{-2}^{4} \left(y^2 + y - \tfrac{1}{2}y^3 + 3y\right) dy$$

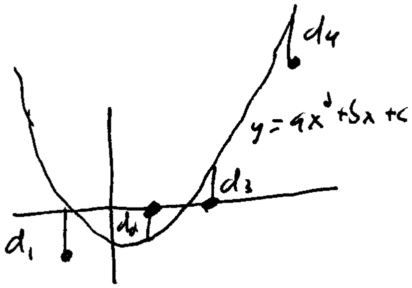
$$= \int_{-2}^{4} \left(-\tfrac{1}{2}y^3 + y^2 + 4y\right) dy$$

$$= -\frac{1}{8}y^4 + \frac{1}{3}y^3 + 2y^2 \Big|_{-2}^{4}$$

$$= -32 + \frac{64}{3} + \cancel{16}\,32 + 2 + \frac{8}{3} - 8$$

$$= 18$$

3. Set up, but **do not solve**, the following problem:
Find the parabola $ax^2 + bx + c$ that best fits the data points (1, 0), (2, 0), (4, 10), and (-1, -3).
(That is, write down the function to be optimized, but don't optimize it.)



Minimize $d_1^2 + d_1^2 + d_3^2 + d_4^2 = \cancel{(a-b+c+3)^2 + (4a+2b+c)^2 + 1}$

$$= (a-b+c+3)^2 + (a+b+c)^2 + (4a+2b+c)^2 + (16a+4b+c-10)^2$$

$$= f(a, b, c)$$

4. Find the region R for which the triple integral

$$\iiint_R (1 - x^2 - y^2 - z^2)\, dV$$

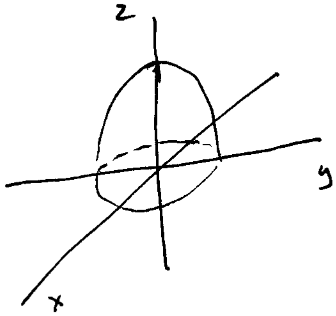
is a maximum.

Want R to include all the pts where $1-x^2-y^2-z^2$ is ≥ 0, and none of the pts. where it's negative:

$$1-x^2-y^2-z^2 \geq 0 \iff x^2+y^2+z^2 \leq 1$$

Thus R is the solid ball of radius 1.

5. Set up, but **do not evaluate**, an *iterated* integral giving the average value of the function $f(x,y,z) = x^2z + y^2z$ over the region enclosed by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$.



In rectangular coordinates, $\iiint_R f\,dV$ is

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^{1-x^2-y^2} (x^2z+y^2z)\,dz\,dy\,dx$$

or

$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_0^{1-x^2-y^2} (x^2z+y^2z)\,dz\,dx\,dy$$

Better in cylindrical coordinates

$$\int_0^{2\pi} \int_0^1 \int_0^{1-r^2} r^2 z r\,dz\,dr\,d\theta$$

or

$$\int_0^1 \int_0^{2\pi} \int_0^{1-r^2} r^2 z r\,dz\,d\theta\,dr$$

So the average value of f over R is $\dfrac{\iiint_R f\,dV}{\iiint_R dV}$, where we evaluate the triple integrals as above.

EXTRA CREDIT Draw the monkey in his saddle.