E11 Exam 3, 2006 – In Class

Name: ANSWER (2 pts)

Do all three problems, all count equally towards your grade. Some parts are much easier than others, if you get stuck, move on so you have a chance to answer all of the easier parts.

Problem 1 Short Answer. Answer the following questions very briefly, preferably in one or two sentences. I am looking for a general understanding, not subtleties.

a) Describe what the homogeneous response, natural response and complete response represent. Use no more than one short sentence for each.

homogeneous - natural response of circuit → form determined by circuit. Magnitude determined by i.c.'s + I.inp.
particular - response to input after transients have died out.
complete = homogeneous + particular - response to an input + i.c.'s for t > 0

b) Why do power companies like to have a power factor close to unity?

If power factor is small, there is excess current in power lines (loss energy) that is going to charging & discharging unreactive load but doing no useful work.

c) Impulses are hard to create in the lab. Why, then, do we care about the response of a circuit to an impulse (i.e., the impulse response)?

We can approximate any input by a series of impulses → output would be a sum of copies of impulse responses.
d) For an inductor, $v = L \frac{di}{dt}$. What does this tell you about the voltage at low frequencies? What is the impedance of an inductor at low frequencies ($\omega \to 0$)?

\[
\frac{di}{dt} \text{ is small } \Rightarrow v \to 0
\]

$Z \to 0$ as $\omega \to 0$ ($\omega \to 0$)

e) Why do we use phasors?

allows us to easily calculate $5.55$.

$V(t) = V(0) \cos(\omega t - \phi) e^{-t/\tau}$

particular homogeneous

f) What does the imaginary part of complex power represent?

- It goes to charging (and discharging) a reactive impedance.

If there is a discontinuity in the step response (\( \uparrow \)), there will be an impulse in the impulse response (\( \uparrow \))
Problem 2 Phasors

Consider the circuit shown. We have used this in class and we know that the voltage across the resistor is a maximum when \( \omega = \omega_0 = \frac{1}{\sqrt{LC}} = 1000 \text{ rad/sec} \). Let's explore its behavior at this frequency.

a) What is the impedance of each element (resistor, capacitor, and inductor) at \( \omega = 1000 \text{ rad/sec} \)? Express in both polar (magnitude and phase) and rectangular (real + imaginary) form.

\[
\begin{align*}
Z_R & \quad R = 1 \quad = 1 \angle 0 \\
Z_L & \quad j\omega L = j10 \quad = 10 \angle 90^\circ \\
Z_C & \quad \frac{1}{j\omega C} = -j10 \quad = 10 \angle -90^\circ
\end{align*}
\]

b) If \( v_i(t) = \cos(1000t) \), what is the current through the circuit (flowing clockwise)? Express both as a phasor (in polar form), and as a function of time.

\[
\begin{align*}
V_i & = 1 \angle 0 \\
I & = \frac{V_i}{Z} = \frac{1 \angle 0}{1 + j10 - j10} = 1 \angle 0 = \cos(1000t)
\end{align*}
\]
c) Using phasors, find an expression for the voltage across the capacitor, \( V_C \) (expressed as a phasor in polar form). Repeat for the voltage across the resistor, \( V_R \). Repeat for the voltage across the inductor, \( V_L \).

\[
V_C = 10 \angle -90^\circ = 10 \cos(9000t - 90^\circ) = 10 \sin(9000t)
\]

\[
V_R = 1 \angle 0^\circ = \cos(1000t)
\]

\[
V_L = 10 \angle -90^\circ = 10 \cos(10000t + 90^\circ) = -10 \sin(10000t)
\]

See part c.

e) Roughly sketch a phasor diagram showing four phasors: the input voltage, the capacitor voltage, the resistor voltage and the inductor voltage. Does your diagram show that $V_R + V_C + V_L = V_{in}$?

$f) $ Roughly sketch, as a function of time, four voltages: $V_{in}, V_C, V_R$, and $V_L$. Does your diagram show that $V_R + V_C + V_L = V_{in}$?
Problem 3 Complex Power

A coil draws 1 Amp peak current at a 0.6 lagging power factor from a 120 V-rms 60 Hz source. Assume that the coil is modeled by a series RL circuit (where R represents the resistance of the coil).

a) What are the apparent power, reactive power, and real power?

\[
\text{Apparent Power} = \frac{V \cdot I}{\sqrt{2}} = 120 \cdot I_{\text{rms}} \cdot V_{\text{rms}} = 84.8 \text{ VA}
\]

\[
\text{Real Power} = 84.8 \cdot 0.6 = 51 \text{ W}
\]

\[
\text{Reactive Power} = 84.8 \cdot \sin(53) = 84.8 \cdot 0.8 = 67.8 \text{ VAR}
\]

b) What is the resistance of the coil?

\[
S = 51 + j67.8 = V \cdot I^* = I_{\text{rms}}^2 Z
\]

\[
= \left(\frac{1}{\sqrt{2}}\right)^2 (R + j\omega L)
\]

\[
R = 51 \quad R = 102 \Omega
\]

c) What is the inductance of the coil?

\[
\frac{\omega L}{2} = 67.8 \quad L = 0.36 \text{ H}
\]

d) What size capacitor (in Farads) is needed to bring the power factor to unity?

\[
P_{\text{cap}} = \frac{V^2}{Z_{\text{cap}}} = \frac{V^2}{\left(\frac{1}{j\omega C}\right)} = -V^2 j\omega C = -67.8 \text{ VA} \text{r.e.}
\]

\[
C = 12.5 \mu \text{F}
\]
e) Calculate the impedance of the load both before the capacitor is added.

\[ Z = R + j \omega L = 102 + j 132 \]

f) Calculate the impedance of the load after the capacitor is added.

\[ Z = \frac{(R + j \omega L) \frac{1}{j \omega C}}{R + j \omega L + \frac{1}{j \omega C}} = \frac{(102 + 132) (-j 212)}{102 + j 132 - 1212} = \frac{28,632 - j 21624}{102 - 76.2} \]

\[ = \frac{36,040 \angle -36.8}{127 \angle -36.8} = 283 \Omega \]

\[ I = \frac{120 \text{ V}_{\text{rms}}}{Z} = 0.42 \text{ A}_{\text{rms}} = 0.6 \text{ A} \text{ peak} \]

**Hard Way**

\[ R = \frac{V^2}{I} = \frac{120^2}{51} = 283 \]

**Easy Way**
This exam is due by 4:00 p.m. Tuesday in my office. You may use your notes, your textbook (only Nilsson and Riedel), and a computer (e.g. Matlab – this will not be necessary, but you may use it if desired) and the course web page (but no other websites or texts).

Problem 4 Complete Response.

For the circuit shown, the output is the voltage across the inductor. The switch moves to the upper position (i.e., it connects the source to R) at t=0.

a) What is the form of the homogeneous response?

\[ v_{o,h} = A e^{-\frac{t}{\tau}} = Ae^{-\frac{t}{\frac{1}{2}}} = Ae^{-2t} \]

b) If the source is 2\cos(4t), what is the particular response (sinusoidal steady state)?

\[ \frac{V_i}{V_s} = \frac{5L}{5L+R} = \frac{1}{1+j\omega L} = \frac{\frac{1}{1+j\frac{\omega}{L}}}{\frac{1}{1+j\frac{\omega}{L}}} = \frac{1}{1+j\frac{\omega}{L}} = 0.97 \angle 14^\circ \]

\[ V_{p,t} = 2 \cos(4t) \cos(4t + 14^\circ) = 1.97 \cos(4t - 14^\circ) \]
c) What is the complete response? Sketch (or plot) it on another piece of paper, and include it after this sheet. Plot for 10 seconds. Make sure $u_0 + v_0$ are clearly labelled.

\[ u_{0,e} = Ae^{-t} + 1.94 \cos(4t + 14.1^\circ) \]
\[ A = 0.12 \]

\[ u_0 = 1.94 \cos(4t + 14.1^\circ) + 0.12e^{-t} \]

Plot on last page.
We would like to eliminate the contribution from the homogeneous response (i.e., we want its magnitude to be zero). If we are able to adjust the timing of when we move the switch relative to the source, we can consider the source to be $2\cos(4t+\phi)$.

d) What is $\phi$ such that the magnitude of the homogeneous response is zero?

\[ \phi = \arccos \left( \frac{\cos(\theta) - 1}{\sin(\theta)} \right) = -1^\circ \]
e) What is the complete response? Sketch (or plot) it on another piece of paper, and include it after this sheet.

Plot for 10 seconds. Make sure $v_i$ and $v_o$ are clearly labelled.

**Solution**

\[ v_0(t) = 1.78 \cos (4t - 90°) = 1.78 \sin (4t) \]

**Caution**

\[ v_0(t) = 1.78 \cos (4t - 90°) = 1.78 \sin (4t) \]

Plot on last page

\[ v_i(t) = 2 \cos (4t - 14°) = 2 \cos (4t + \phi) \]

\[ v_o(t) = 1.94 \cos (4t) = 1.94 \cos (4t + \phi + \Theta) \]
f) Clearly we can eliminate the homogeneous response in a first order circuit simply by closing the switch at a particular phase of the input voltage. Do you think this would be possible if the circuit were second order? Briefly defend your answer.

No ⇒ we only have one parameter to vary (φ) so we can only set one parameter in homogeneous.

Consider

\[ V_{in} = \cos (\omega t + \phi) \]

\[ V_{out} = B \cos (\omega t + \phi + \theta) \]

\[ V_{out,h} = A_1 e^{s_1 t} + A_2 e^{s_2 t} \] overdumped for easy solution.

this is the only parameter we control

\[ V_{out} = B \cos (\omega t + \phi + \theta) + A_1 e^{s_1 t} + A_2 e^{s_2 t} \]

1. \[ V_{out}(0) = B \cos (\phi + \theta) + A_1 + A_2 = 0 \]

\[ \text{use zero i.c.'s} \]

2. \[ \frac{dV_{out}(0)}{dt} = B \omega \sin (\phi + \theta) + A_1 s_1 + A_2 s_2 = 0 \]

we want \( A_1 = A_2 = 0 \) to eliminate homogeneous response.

From ① \( B \cos (\phi + \theta) = 0 \) can't do both.

From ② \( B \omega \sin (\phi + \theta) = 0 \)
Take Home, Exam 3, 2006

R=1; L=1; w=4;
t=linspace(0,5);

part b

H=j*w*L/(R+j*w*L);
M=abs(H); theta=angle(H);
disp(['M = ' num2str(M)])
disp(['theta = ' num2str(theta*180/pi)])

M = 0.97014
theta = 14.0362

part c

vi=2*cos(4*t);
A=2-2*M*cos(theta);
disp(['A = ' num2str(A)])
vh=A*exp(-t); %homogeneous
vp=2*M*cos(w*t+theta); %particular
vc=vh+vp; %complete
plot(t,vi, '-.', t,vh);
legend('Vin', 'Vout');
title('Part c, take home');

A = 0.11765
part d

```matlab
syms phi
solve('2*M*cos(phi+theta)-2*cos(phi)', 'phi')
phi=eval(ans);
disp(['phi = ', num2str(phi*180/pi)]);
```

ans =
atan((M*cos(theta)-1)/M/sin(theta))

\( \phi = -14.0362 \)

part e

```matlab
vi=2*cos(4*t+phi);
A=2*cos(phi)-2*M*cos(theta+phi);
disp(['A = ', num2str(A)])
vh=A*exp(-t); %homogeneous
vp=2*M*cos(w*t+theta+phi); %particular
vc=vh+vp; %complete
plot(t,vi,'--',t,vh);
legend('vin','vout');
title('Part e, take home');
```

\[ A = 0 \]