Clarification/Correction of E11 class notes, 11/22/06.

In class I made several errors when developing the Bode Plot of the final example, and I only quickly explained them. These notes should help clarify the corrections.

The task was to plot the asymptotic Bode Plot of \( H(s) = \frac{s^2 + 11s + 10}{s^3 + 10s^2 + 10000s} \)

Factor numerator and denominator as much as possible:
\[
H(s) = \frac{(s+1)(s+10)}{s(s^2+10s+10000)}
\]

Note: denominator is not factored further because roots are complex.

Put into standard Bode form
\[
\frac{10 \left( \frac{s+1}{1} \right) \left( \frac{s+10}{10} \right)}{10000s \left( \frac{s^2}{10000} + \frac{10}{10000} \frac{s+1}{s+1} \right) + 1} = \frac{1}{1000} \left( \frac{s+1}{1} \right) \left( \frac{s+10}{10} \right) \left( \frac{s}{s+10000} \right) + 0.1 \left( \frac{s}{100} \right) + 1
\]

To find \( \omega_n \) and \( \zeta \) for 2\textsuperscript{nd} order term in denominator, compare to standard form:
\[
\left( \frac{s}{100} \right)^2 + 0.1 \left( \frac{s}{100} \right) + 1 = \left( \frac{s}{\omega_n} \right)^2 + 2\zeta \left( \frac{s}{\omega_n} \right) + 1
\]

Clearly \( \omega_n=100 \) and \( \zeta=0.05 \).

(Likewise, we could have determined \( \omega_n \) and \( \zeta \) from original representation: \( s^2 + 10s + 10000 = s^2 + 2\zeta\omega_n s + \omega_n^2 \))

Now we can separate our transfer function
\[
H(s) = \frac{1}{1000} \left( \frac{s+1}{1} \right) \left( \frac{s+10}{10} \right) \left( \frac{s}{s+10000} \right) + 0.1 \left( \frac{s}{100} \right) + 1
\]

into its constituent parts.

It has:

- a constant multiplier of \( 1/1000 = 0.001 = -60\text{dB} \)
- a real zero at \( s=-1 \)
- a real zero at \( s=-10 \)
- a pole at the origin
- complex conjugate poles with \( \omega_n=100 \) and \( \zeta=0.05 \)

Bode plot (from BodePlotGui is on next page).

```plaintext
>> MySys=tf([1 11 10],[1 10 10000 0])
Transfer function:
s^2 + 11 s + 10
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s^3 + 10 s^2 + 10000 s
>> BodePlotGui(MySys)
```