

## Common Laplace Transform Pairs

Time Domain Function		Laplace Domain Function
Name	Definition*	
Unit Impulse	$\delta(t)$	1
Unit Step	$u(t)$	$\frac{1}{s}$
Unit Ramp	t	$\frac{1}{s^2}$
Exponential	$e^{-at}$	$\frac{1}{s+a}$
Asymptotic Exponential	$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
Dual Exponential	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
Asymptotic Dual Exponential	$\frac{1}{ab} \left[ 1 + \frac{1}{a-b}(be^{-at} - ae^{-bt}) \right]$	$\frac{1}{s(s+a)(s+b)}$
Time multiplied Exponential	$te^{-at}$	$\frac{1}{(s+a)^2}$
Sine	$\sin(\omega_n t)$	$\frac{\omega}{s^2 + \omega_n^2}$
Cosine	$\cos(\omega_n t)$	$\frac{s}{s^2 + \omega_n^2}$
Decaying Sine	$e^{-at} \sin(\omega_n t)$	$\frac{\omega}{(s+a)^2 + \omega_n^2}$
Decaying Cosine	$e^{-at} \cos(\omega_n t)$	$\frac{s+a}{(s+a)^2 + \omega_n^2}$
Generic Oscillatory Decay	$e^{-at} \left[ B \cos(\omega_n t) + \frac{C - aB}{\omega_n} \sin(\omega_n t) \right]$	$\frac{Bs + C}{(s+a)^2 + \omega_n^2}$
Prototype Second Order Lowpass, underdamped	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t)$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
Prototype Second Order Lowpass, underdamped - Step Response	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t + \phi)$ $\phi = \tan^{-1} \left( \frac{\sqrt{1-\zeta^2}}{\zeta} \right)$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$

\*All time domain functions are implicitly=0 for t<0 (i.e. they are multiplied by unit step).

## Common Laplace Transform Properties

Name	Illustration
Definition of Transform	$f(t) \xrightarrow{L} F(s)$ $F(s) = \int_{0^-}^{\infty} f(t)e^{-st} dt$
Linearity	$Af_1(t) + Bf_2(t) \xrightarrow{L} AF_1(s) + BF_2(s)$
First Derivative	$\frac{df(t)}{dt} \xrightarrow{L} sF(s) - f(0^-)$
Second Derivative	$\frac{d^2 f(t)}{dt^2} \xrightarrow{L} s^2 F(s) - sf(0^-) - \dot{f}(0^-)$
$n^{\text{th}}$ Derivative	$\frac{d^n f(t)}{dt^n} \xrightarrow{L} s^n F(s) - \sum_{i=1}^n s^{n-i} f^{(i-1)}(0^-)$
Integral	$\int_0^t f(\lambda) d\lambda \xrightarrow{L} \frac{1}{s} F(s)$
Time Multiplication	$tf(t) \xrightarrow{L} -\frac{dF(s)}{ds}$
Time Delay	$f(t-a)U(t-a) \xrightarrow{L} e^{-as}F(s)$
Complex Shift	$f(t)e^{-at} \xrightarrow{L} F(s+a)$
Scaling	$f\left(\frac{t}{a}\right) \xrightarrow{L} aF(as)$
Convolution Property	$f_1(t) * f_2(t) \xrightarrow{L} F_1(s)F_2(s)$
Initial Value	$\lim_{t \rightarrow 0^+} f(t) = \lim_{s \rightarrow \infty} sF(s)$
Final Value (if final value exists)	$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$