

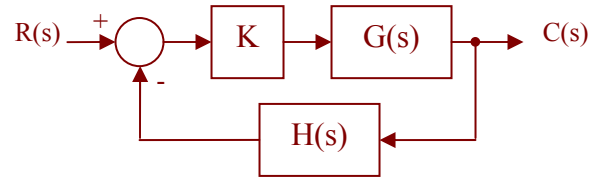
# Rules for Making Root Locus Plots

The closed loop transfer function of the system shown is

$$T(s) = \frac{KG(s)}{1 + KG(s)H(s)}$$

So the characteristic equation (*c.e.*) is

$$1 + KG(s)H(s) = 1 + K \frac{N(s)}{D(s)} = 0, \text{ or } D(s) + K N(s) = 0.$$



As  $K$  changes, so do locations of closed loop poles (i.e., zeros of *c.e.*). The table below gives rules for sketching the location of these poles for  $K=0 \rightarrow \infty$  (i.e.,  $K \geq 0$ ).

Rule Name	Description
<b>Definitions</b>	<ul style="list-style-type: none"> <li>The loop gain is <math>KG(s)H(s)</math> or <math>K \frac{N(s)}{D(s)}</math>.</li> <li><math>N(s)</math>, the numerator, is an <math>m^{\text{th}}</math> order polynomial; <math>D(s)</math>, is <math>n^{\text{th}}</math> order.</li> <li><math>N(s)</math> has zeros at <math>z_i</math> (<math>i=1..m</math>); <math>D(s)</math> has them at <math>p_i</math> (<math>i=1..n</math>).</li> <li>The difference between <math>n</math> and <math>m</math> is <math>q</math>, so <math>q=n-m</math>. (<math>q \geq 0</math>)</li> </ul>
<b>Symmetry</b>	The locus is symmetric about real axis (i.e., complex poles appear as conjugate pairs).
<b>Number of Branches</b>	There are $n$ branches of the locus, one for each closed loop pole.
<b>Starting and Ending Points</b>	The locus starts ( $K=0$ ) at poles of loop gain, and ends ( $K \rightarrow \infty$ ) at zeros. Note: this means that there will be $q$ roots that will go to infinity as $K \rightarrow \infty$ .
<b>Locus on Real Axis*</b>	The locus exists on real axis to the left of an odd number of poles and zeros.
<b>Asymptotes as <math> s  \rightarrow \infty</math>*</b>	If $q > 0$ there are asymptotes of the root locus that intersect the real axis at $\sigma = \frac{\sum_{i=1}^m p_i - \sum_{i=1}^n z_i}{q}$ , and radiate out with angles $\theta = \pm r \frac{180}{q}$ , where $r=1, 3, 5 \dots$
<b>Break-Away/-In Points on Real Axis</b>	Break-away or -in points of the locus exist where $N(s)D'(s) - N'(s)D(s) = 0$ .
<b>Angle of Departure from Complex Pole*</b>	Angle of departure from pole, $p_j$ is $\theta_{\text{depart}, p_j} = 180^\circ + \sum_{i=1}^m \angle(p_j - z_i) - \sum_{\substack{i=1 \\ i \neq j}}^n \angle(p_j - p_i)$ .
<b>Angle of Departure at Complex Zero*</b>	Angle of arrival at zero, $z_j$ , is $\theta_{\text{arrive}, z_j} = 180^\circ + \sum_{\substack{i=1 \\ i \neq j}}^m \angle(z_j - z_i) - \sum_{i=1}^n \angle(z_j - p_i)$ .
<b>Locus Crosses Imaginary Axis</b>	Use Routh-Hurwitz to determine where the locus crosses the imaginary axis.
<b>Given Gain "K," Find Poles</b>	Rewrite <i>c.e.</i> as $D(s) + KN(s) = 0$ . Put value of $K$ into equation, and find roots of <i>c.e.</i> (This may require a computer)
<b>Given Pole, Find "K."</b>	Rewrite <i>c.e.</i> as $K = -\frac{D(s)}{N(s)}$ , replace "s" by desired pole location and solve for $K$ . Note: if "s" is not exactly on locus, $K$ may be complex (small imaginary part). Use real part of $K$ .

\*These rules change to draw complementary root locus ( $K \leq 0$ ). See next page for details.

## Complementary Root Locus

To sketch complementary root locus ( $K \leq 0$ ), most of the rules are unchanged except for those in table below.

Rule Name	Description
<b>Locus on Real Axis</b>	The locus exists on real axis to the right of an odd number of poles and zeros.
<b>Asymptotes as <math> s  \rightarrow \infty</math></b>	If $q > 0$ there are asymptotes of the root locus that intersect the real axis at $\sigma = \frac{\sum_{i=1}^m p_i - \sum_{i=1}^n z_i}{q}$ , and radiate out with angles $\theta = \pm p \frac{180}{q}$ , where $p=0, 2, 4, \dots$
<b>Angle of Departure from Complex Pole</b>	Angle of departure from pole, $p_j$ is $\theta_{\text{depart}, p_j} = \sum_{i=1}^m \angle(p_j - z_i) - \sum_{\substack{i=1 \\ i \neq j}}^n \angle(p_j - p_i).$
<b>Angle of Departure at Complex Zero</b>	Angle of arrival at zero, $z_j$ , is $\theta_{\text{arrive}, z_j} = \sum_{\substack{i=1 \\ i \neq j}}^m \angle(z_j - z_i) - \sum_{i=1}^n \angle(z_j - p_i).$

## Other Forms of Root Locus

Not yet complete.

## Design with Root Locus

Not yet complete.