

WEEK 8

$$I_1 = g_{11}V_1 + g_{12}I_0$$

$$V_2 = g_{21}V_1 + g_{22}I_0$$

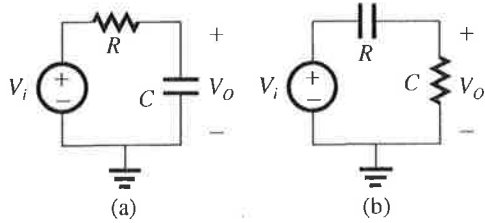
thus

$$\left. \frac{V_2}{I_2} \right|_{v_1=0} = g_{22} = R_0 \quad \left. \frac{I_1}{V_1} \right|_{I_2=0} = g_{11} = \frac{1}{R_i}$$

$$\left. \frac{V_2}{I_2} \right|_{I_0=0} = g_{21} = Av \quad \left. \frac{I_2}{I_1} \right|_{v_1=0} = g_{12} = \infty$$

due to unilateral nature of Figure 1.16a

1.65



for (a) $V_o = V_i \left(\frac{1/SC}{1/SC + R} \right)$

$$\frac{V_o}{V_i} = \frac{1}{1 + SCR}$$

where $k=1$

$$\omega_0 = \frac{1}{RC} \text{ from table 1.2 it is low pass.}$$

for (b) $V_o = V_i \left(\frac{R}{R + \frac{1}{SC}} \right)$

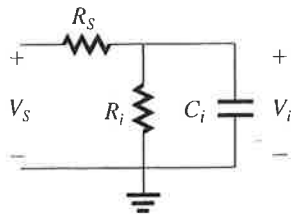
$$\frac{V_o}{V_i} = \frac{SRC}{1 + SCR}$$

$$\frac{V_o}{V_i} = \frac{S}{S + \frac{1}{RC}}$$

where $k=1$

$$\omega_0 = \frac{1}{RC} \text{ from table 1.2 it is high pass.}$$

1.66



$$\frac{V_i}{V_s} = \frac{\frac{R_i \frac{1}{sC_i}}{R_i + \frac{1}{sC_i}}}{R_s + \left(\frac{R_i \frac{1}{sC_i}}{R_i + \frac{1}{sC_i}} \right)} = \frac{R_i}{1 + sC_i R_i} \cdot \frac{1}{R_s + \left(\frac{R_i}{1 + sC_i R_i} \right)}$$

$$= \frac{R_i}{R_s + sC_i R_i R_s + R_i}$$

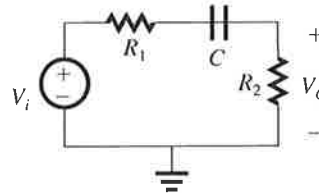
$$\frac{V_i}{V_s} = \frac{R_i}{(R_s + R_i) + sC_i R_i R_s} = \frac{\frac{R_i}{(R_s + R_i)}}{1 + S \left(\frac{C_i R_i R_s}{R_s + R_i} \right)}$$

Where $K = \frac{R_i}{(R_s + R_i)}$

$$\omega = \frac{R_s + R_i}{C_i R_i R_s} \text{ from table 1.2 low pass for given}$$

values $\omega_0 = 12.5 \text{ MHz}$

1.67 Using the voltage-divider rule.



$$T(s) = \frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1 + \frac{1}{sC}}$$

$$T(s) = \left(\frac{R_2}{R_1 + R_2} \right) \left(\frac{s}{s + \frac{1}{C(R_1 + R_2)}} \right)$$

which is from Table 1.2 is of the high-pass type with

$$K = \frac{R_2}{R_1 + R_2} \quad \omega_0 = \frac{1}{C(R_1 + R_2)}$$

As a further verification that this is a high-pass network and $T(s)$ is a high-pass transfer function, we assume as $s \rightarrow 0$, $T(s) \rightarrow 0$; and as $s \rightarrow \infty$, $T(s) = R_2 / (R_1 + R_2)$. Also, from the circuit

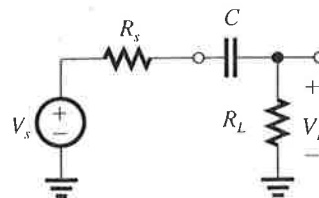
observe as $s \rightarrow \infty$, $(1/sC) \rightarrow 0$ and $V_o/V_i = R_2 / (R_1 + R_2)$. Now, for

$$R_1 = 10 \text{ k}\Omega, R_2 = 40 \text{ k}\Omega, \text{ and } C = 0.1 \mu\text{F}.$$

$$f_o = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \times 0.1 \times 10^{-6} (10 + 40) \times 10^3} = 31.8 \text{ Hz}$$

$$|T(j\omega_0)| = \frac{K}{\sqrt{2}} = \frac{40}{10 + 40} \frac{1}{\sqrt{2}} = 0.57 \text{ V/V}$$

1.68 Using the voltage divider rule,



$$\frac{V_O}{V_S} = \frac{R_L}{R_L + R_S + \frac{1}{sC}}$$

$$= \frac{R_L}{R_L + R_S} \frac{s}{C(R_L + R_S)}$$

which is of the high-pass STC type (see Table 1.2) with

$$K = \frac{R_L}{R_L + R_S} \quad \omega_0 = \frac{1}{C(R_L + R_S)}$$

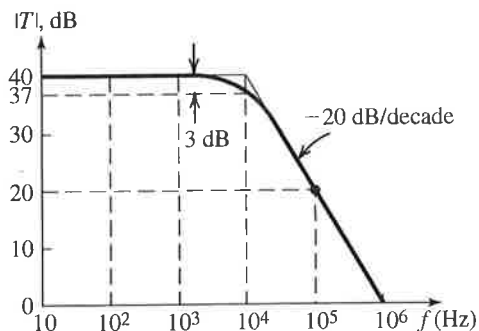
For $f_o \leq 10$ Hz

$$\frac{1}{2\pi C(R_L + R_S)} \leq 10$$

$$\Rightarrow C \geq \frac{1}{2\pi \times 10(20 + 5) \times 10^3}$$

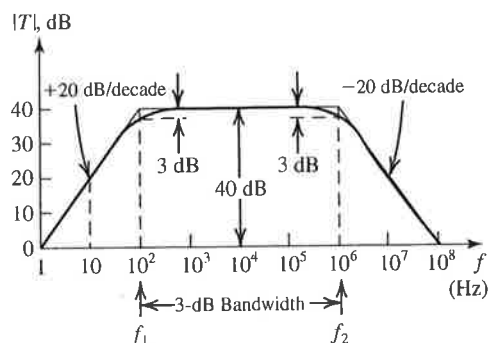
Thus, the smallest value of C that will do the job is $C = 0.64 \mu F$.

1.69 The given measured data indicate that this amplifier has a low-pass STC frequency response with a low-frequency gain of 40 dB, and a 3-dB frequency of 10^4 Hz. From our knowledge of the Bode plots for low-pass STC networks (Figure 1.23a) we can complete the Table entries and sketch the amplifier frequency response



$f(\text{Hz})$	$ T (\text{dB})$	$\angle T(^{\circ})$
0	40	0
100	40	0
1000	40	0
10^4	37	-45°
10^5	20	-90°
10^6	0	-90°

1.70 From our knowledge of the Bode plots of STC low-pass and high-pass networks we see that this amplifier has a mid-band gain of 40 dB, a low-frequency response of the high-pass STC type with $f_{3dB} = 10^2$ Hz, and a high-frequency response of the low-pass STC type with $f_{3dB} = 10^6$ Hz. We thus can sketch the amplifier frequency response and complete the table entries as follows



$f(\text{Hz})$	1	10	10^2	10^3	10^4	10^5	10^6	10^7	10^8
$ T (\text{dB})$	0	20	37	40	40	40	37	20	0

1.71 Since the overall transfer function is that of three identical STC LP circuits in cascade (but with no loading effects since the buffer amplifiers have input and zero output resistances) the overall gain will drop by 3 dB below the value at dc at the frequency for which the gain of each STC circuit is 1 dB down. This frequency is found as follows: The transfer function of each STC circuit is

$$T(s) = \frac{1}{1 + \frac{s}{\omega_0}}$$

where

$$\omega_0 = 1 / CR$$

Thus,

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega_{1dB}}{\omega_0}\right)^2}} = -1$$

$$\Rightarrow 1 + \left(\frac{\omega_{1dB}}{\omega_0}\right)^2 = 10^{0.1}$$

$$\omega_{1dB} = 0.51 \omega_0$$

$$\omega_{1dB} = 0.51 / CR$$

$$c) 1 + \frac{R_2}{R_1} = 2 \text{ V/V}, f_{3\text{db}} = 10 \text{ kHz}$$

$$f_t = 10 \text{ MHz} \times 2 = 20 \text{ MHz}$$

$$d) -\frac{R_2}{R_1} = -2 \text{ V/V}, f_{3\text{db}} = 10 \text{ kHz}$$

$$f_t = 10 \text{ MHz}(1 + 2) = 30 \text{ MHz}$$

$$e) -\frac{R_2}{R_1} = -1000 \text{ V/V}, f_{3\text{db}} = 20 \text{ kHz}$$

$$f_t = 20 \text{ kHz}(1 + 100) = 20.02 \text{ MHz}$$

$$f) 1 + \frac{R_2}{R_1} = 1 \text{ V/V}, f_{3\text{db}} = 1 \text{ MHz}$$

$$f_t = 1 \text{ M} \times 1 = 1 \text{ MHz}$$

$$g) -\frac{R_2}{R_1} = -1, f_{3\text{db}} = 1 \text{ MHz}$$

$$f_t = 1 \text{ M}(1 + 1) = 2 \text{ MHz}$$

2.114

$$\text{Gain} = 1 + \frac{R_2}{R_1} = 96 \text{ V/V}$$

$$f_{3\text{dB}} = 8 \text{ kHz}$$

$$f_t = 96 \times 8 = 768 \text{ kHz}$$

$$\text{for } f_{3\text{dB}} = 24 \text{ kHz}$$

$$\text{Gain} = \frac{768}{24} = 32 \text{ V/V}$$

2.115

$$f_{3\text{db}} = f_t = 1 \text{ MHz}$$

$$|G| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3\text{db}}}\right)^2}} = \frac{1}{\sqrt{1 + f^2}} f_{\text{in}} \text{ MHz}$$

$$|G| = 0.99 \Rightarrow f = 0.142 \text{ MHz}$$

The follower behaves like a low-pass STC circuit

$$\text{with a time constant } \tau = \frac{1}{2\pi \times 10^6} = \frac{1}{2\pi} \mu\text{s}$$

$$t_r = 2.20 = 0.35 \mu\text{s} \text{ (Refer to Appendix F)}$$

2.116

$$1 + \frac{R_2}{R_1} = 10 \text{ V/V}, R_1 = 1 \text{ k}\Omega, R_2 = 9 \text{ k}\Omega$$

If we consider 5τ the time that it takes for the output voltage to reach 99% of its final value, then: $5\tau = 100 \text{ ns} \Rightarrow \tau = 20 \text{ ns}$

$$\tau = \frac{1}{\omega_{3\text{db}}} \Rightarrow \omega_{3\text{db}} = 50 \times 10^6 \Rightarrow f_{3\text{db}} = 7.96 \text{ MHz}$$

$$f_t = \left(1 + \frac{R_2}{R_1}\right) f_{3\text{db}} = 10 \times 7.96 = 79.6 \text{ MHz}$$

2.117

a) Assume two identical stages, each with a gain

$$\text{function: } G = \frac{G_o}{1 + j\frac{\omega}{\omega_1}} = \frac{G_o}{1 + jf/f_1}$$

$$G = \frac{G_o}{\sqrt{1 + \left(\frac{f}{f_1}\right)^2}}$$

$$\text{overall gain of the cascade is } \frac{G_o^2}{\sqrt{1 + \left(\frac{f}{f_1}\right)^2}}$$

The gain will drop by 3db when:

$$1 + \left(\frac{f_{3\text{db}}}{f_1}\right)^2 = \sqrt{2}, \text{ Note } 3\text{db} = 20\log\sqrt{2}$$

$$f_{3\text{db}} = F_1 \sqrt{\sqrt{2} - 1}$$

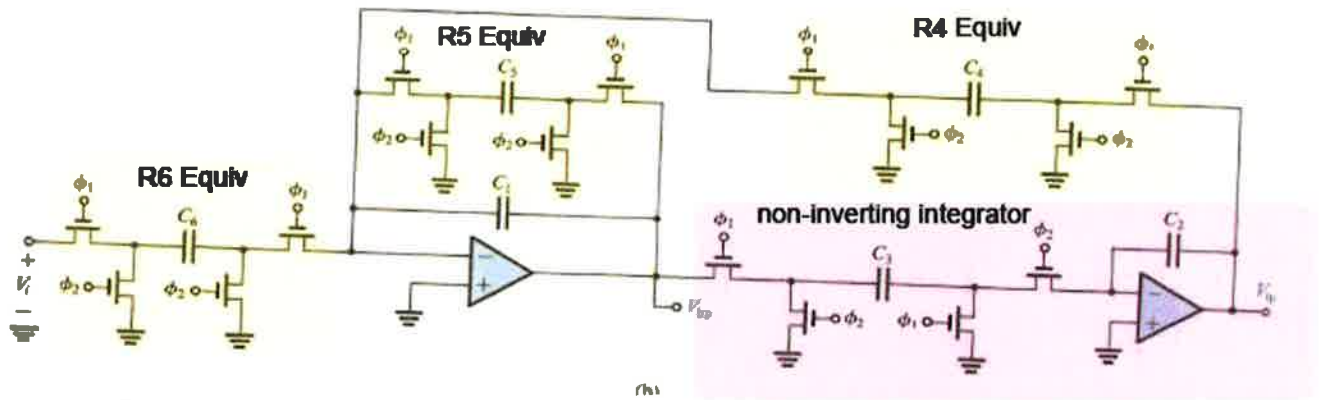
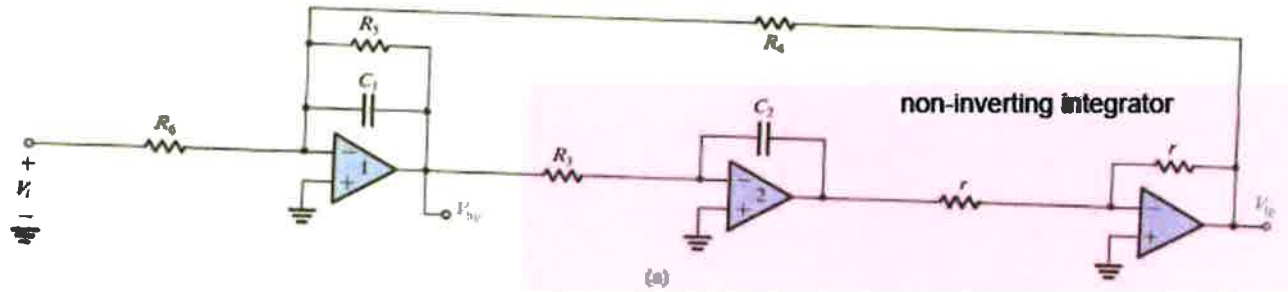
2.118

$f_t = 100 \times 5 = 500 \text{ MHz}$ if single op-amp is used.

with op-amp that has only $f_t = 40 \text{ MHz}$, the possible closed 100 p gain at 5 MHz is:

$$|A| = \frac{40}{5} = 8 \text{ V/V}$$

To obtain an overall gain of 100, three such amplifier cascaded, would be required. Now, if each of the 3 stages, has a low-frequency (d) closed loop gain K, then its 3-db frequency will $\frac{40}{k} \text{ MHz}$.



16.18

(a) $f_p = 3.4 \text{ kHz}$

$A_{\max} = 1 \text{ dB} \Rightarrow t = 0.5088$

$f_s = 4 \text{ kHz} \quad A_{\min} = 35 \text{ dB}$

$\omega_s / \omega_p = 1.176$

Using Eq (16.22):

$$A(\omega_s) = 10 \log \left[1 + t^2 \text{Cosh}^2 \left(N \text{Cosh}^{-1} \left(\frac{\omega_s}{\omega_p} \right) \right) \right]$$

& trying different values for N

$N \quad A(\omega_s)$

8 28.8 dB

9 33.9 dB

10 38.98 dB

\therefore Use $N = 10$

Excess attenuation = $39 - 35 = 4 \text{ dB}$

(b) Poles are given by:

$$P_k = -\omega_p \sin \left(\frac{2k-1}{N} \cdot \frac{\pi}{2} \right) \sinh \left(\frac{1}{N} \left(\sinh^{-1} \left(\frac{1}{t} \right) \right) \right) + j\omega_p \cos \left(\frac{2k-1}{N} \cdot \frac{\pi}{2} \right) \cosh \left(\frac{1}{N} \sinh^{-1} \left(\frac{1}{t} \right) \right)$$

for $k = 1, 2, \dots, N$.

Since $t = 0.5088$ and $N = 10$

$\sinh(1/N \sinh^{-1}(1/t)) = 0.1433$

$\cosh(1/N \sinh^{-1}(1/t)) = 1.010$

$\therefore P_1 = \omega_p \left[-0.1433 \sin \left(\frac{\pi}{20} \right) + j1.010 \cos \left(\frac{\pi}{20} \right) \right]$

$= \omega_p (-0.0224 + j0.9978)$

$P_2 = \omega_p (-0.0650 + j0.900)$

$P_3 = \omega_p (-0.1013 + j0.7143)$

$P_4 = \omega_p (-0.1277 + j0.4586)$

$P_5 = \omega_p (-0.1415 + j0.1580)$

Now it should be realized that the remaining poles are complex conjugates of the above.

pole-pair P_1 & P_1^* give a factor:

$$S^2 + 2(0.0224)\omega_p S + \omega_p^2(0.0224^2 + 0.9978^2) = S^2 + 0.0448\omega_p S + 1.023\omega_p^2$$

i.e. this factor is from $(S - P_1)(S - P_1^*)$

$P_{2 \text{ yields:}} S^2 + 0.130\omega_p S + 0.902\omega_p^2$

$P_{3 \text{ yields:}} S^2 + 0.203\omega_p S + 0.721\omega_p^2$

$P_{4 \text{ yields:}} S^2 + 0.255\omega_p S + 0.476\omega_p^2$

$P_{5 \text{ yields:}} S^2 + 0.283\omega_p S + 0.212\omega_p^2$

Now $T(S)$ is given by

$$T(S) = \frac{k\omega_p^{10}}{E2^9(S - P_1)(S - P_1^*) \dots (S - P_5)(S - P_5^*)}$$

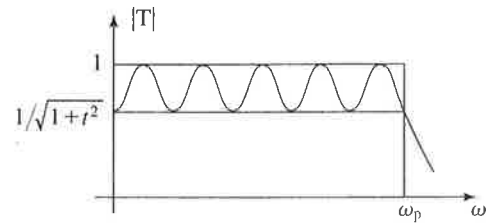
where the second order terms of the denominator are given above.

k is the dc gain

\therefore we want the dc gain to be

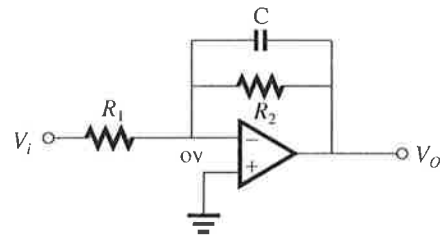
$$k = \frac{1}{1 + t^2} = 0.8913$$

$\omega_p = 2\pi \times 3400$



16.19 $f_o = 10 \text{ KHz}$ DC gain = 10

$R_{in} = 10 \text{ k}\Omega$



$R_{in} = R_1 = 10 \text{ k}\Omega$

DC gain = $-R_2/R_1 = -10$

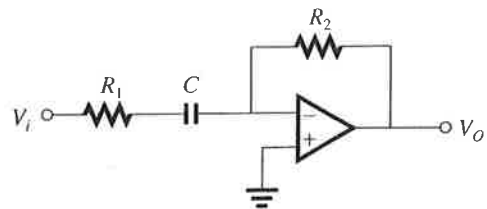
$R_2 = 10R_1 = 100 \text{ k}\Omega$

$R_2 C = 1/W_o$

$$C = \frac{1}{W_o R_2} = \frac{1}{2\pi \cdot 10^4 \times 100 \times 10^3} = 0.159 \text{ nF}$$

16.20 $f_o = 100 \text{ KHz}$ $R_i(\infty) = 100 \text{ k}\Omega$

$|T(\infty)| = 1$



$R_i(\infty) = R_1 = 100 \text{ k}\Omega$

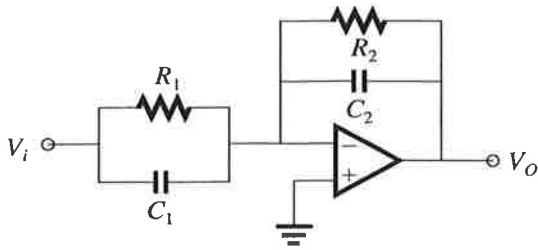
$|T(\infty)| = R_2/R_1 = 1$

$R_2 = R_1 = 100 \text{ k}\Omega$

$C R_1 = 1/W_o$

$$C = \frac{1}{W_o R_1} = \frac{1}{2\pi \cdot 100 \times 10^3 \times 100 \times 10^3} = 15.9 \text{ nF}$$

16.21 Use general first-order circuit:



-Zero at 1 kHz; Pole at 100 kHz

$-|T(O)| = 1; R_i(O) = 1 \text{ k}\Omega$

Thus: $R_i(\text{DC}) = R_1 = 1 \text{ k}\Omega$

$T(\text{DC}) = -R_2/R_1 = -1$

$R_2 = R_1 = 1 \text{ k}\Omega$

For a pole at 100 kHz

$$C_2 R_2 = \frac{1}{W_o} \Rightarrow C_2 = \frac{1}{2\pi f_o R_2}$$

$$= 1.59 \text{ nF}$$

For the circuit $T(S) = \frac{a_1 S + a_0}{S + W_o}$

Thus the Zero at $-a_0/a_1 = -2\pi \cdot 10^3$

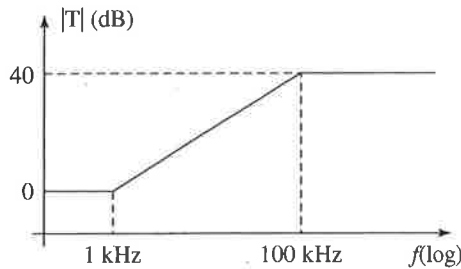
$$C_1 R_1 = a_1/a_0$$

$$C_1 = \frac{1}{2\pi \cdot 10^3 R_1}$$

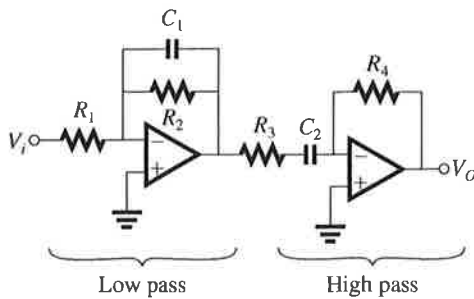
$$= 159 \text{ nF}$$

High freq gain = $\frac{-C_1}{C_2} = -100$

= 40 dB



16.22



$$\text{gain} = 10^{12/20} = 3.98 \approx 4$$

want $R_i = R_1$ large

$$\therefore R_1 = 100 \text{ k}\Omega$$

Total gain = $A_{LP} A_{HP} = 4$

$$A_{LP} = -R_2/R_1 \Rightarrow R_2 = -A_{LP} R_1 \text{ and}$$

$$R_2 \leq 100 \text{ k}\Omega$$

$$\therefore \text{make } A_{LP} = -1 A_{HP} = -4$$

$$R_2 = 100 \text{ k}\Omega$$

$$R_2 C_1 = \frac{1}{W_{o, LP}}$$

$$C_1 = \frac{1}{2\pi f_{o, LP} R_2} = \frac{1}{2\pi (10 \times 10^3) 100 \times 10^3}$$

$$= 0.159 \text{ nF}$$

$$A_{HP} = \left. \begin{array}{l} -R_4/R_3 = -4 \\ R_4 = 4R_3 \end{array} \right\} \text{make } \begin{array}{l} R_4 = 100 \text{ k}\Omega \\ R_3 = 25 \text{ k}\Omega \end{array}$$

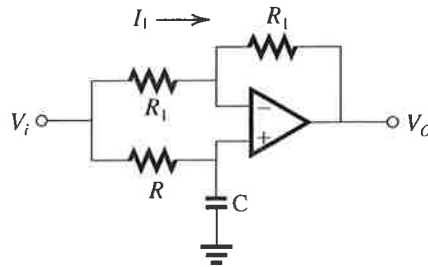
Now $R_3 C_2 = 1/W_{o, HP}$

$$C_2 = \frac{1}{2\pi f_{o, HP} R_3}$$

$$= \frac{1}{2\pi (100 \times 10^3) 25 \times 10^3}$$

$$= 63.7 \text{ nF}$$

16.23



At +ve terminal

$$V_1 = \frac{1/SC}{1/SC + R_1} V_i$$

$$= \frac{1}{1 + S\tau} V_i \quad \tau = R_1 C$$

$V_- = V_+$ due to virtual short between terminals

$$\therefore I_1 = \left(V_i - \frac{1}{1 + S\tau} V_i \right) \frac{1}{R_1}$$

$$V_o = V_2 - I_1 R_1$$

$$= \frac{V_i}{1 + S\tau} - \left(V_i - \frac{V_i}{1 + S\tau} \right) \frac{R_1}{R_1}$$

$$\frac{V_o}{V_i} = \frac{1 - (1 + S\tau) + 1}{1 + S\tau} = \frac{1 - S\tau}{1 + S\tau}$$

$$\phi(\omega) = 180^\circ + \tan^{-1}\left(\frac{\omega}{-\omega_0}\right) - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$\text{sub: } \tan\left(\frac{\omega}{-\omega_0}\right) = 180 - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$\Rightarrow \phi(\omega) = -2 \tan^{-1}(\omega / \omega_0)$$

Now $\phi = -120^\circ$ at $\omega = 2\pi 60$

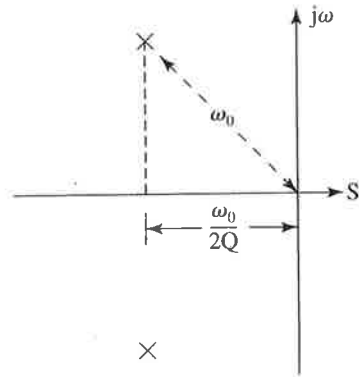
$$-120^\circ = -2 \tan^{-1}(WRC)$$

$$-60 = -\tan^{-1}(2\pi 60 \times R \times 10^{-6})$$

$$R = 4.59 \text{ k}\Omega$$

R_1 can be arbitrarily chosen use $R_1 = 10 \text{ k}\Omega$

16.28



Natural Modes:

$$-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

$$\omega_0 = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$$

$$\frac{\omega_0}{2Q} = \frac{1}{2} \Rightarrow \frac{\omega_0}{Q} = 1$$

$$T(s) = \frac{a_2 s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} = \frac{a_2 s^2}{s^2 + s + 1}$$

$$|T(j\infty)| = a_2 = 1$$

$$\therefore T(s) = \frac{s^2}{s^2 + s + 1}$$

$$\omega_0 = 1$$

$$Q = 1$$

16.29

For a 2nd-order bandpass

$$T(s) = \frac{a_1 s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$T(j\omega) = \frac{j\omega a_1}{(\omega_0^2 - \omega^2) + \frac{j\omega\omega_0}{Q}}$$

$$|T(j\omega)| = \frac{a_1 \omega}{\left[(\omega_0^2 - \omega^2)^2 + \frac{\omega^2 \omega_0^2}{Q^2} \right]^{1/2}}$$

Part (a):

$$|T(j\omega_1)| = |T(j\omega_2)|$$

$$\frac{a_1 \omega_1}{\sqrt{(\omega_0^2 - \omega_1^2)^2 + \left(\frac{\omega_1 \omega_0}{Q}\right)^2}}$$

$$= \frac{a_1 \omega_2}{\sqrt{(\omega_0^2 - \omega_2^2)^2 + \left(\frac{\omega_2 \omega_0}{Q}\right)^2}}$$

$$\omega_1^2 \left[(\omega_0^2 - \omega_2^2)^2 + \left(\frac{\omega_2 \omega_0}{Q}\right)^2 \right]$$

$$= \omega_2^2 \left[(\omega_0^2 - \omega_1^2)^2 + \left(\frac{\omega_1 \omega_0}{Q}\right)^2 \right]$$

$$\omega_1^2 (\omega_0^4 - 2\omega_0^2 \omega_2^2 + \omega_2^4) = \omega_2^2 (\omega_0^4 - 2\omega_0^2 \omega_1^2 + \omega_1^4)$$

$$\omega_1^2 \omega_0^4 + \omega_1^2 \omega_2^4 = \omega_2^2 \omega_0^4 + \omega_2^2 \omega_1^4$$

$$\omega_0^4 (\omega_1^2 - \omega_2^2) = \omega_2^2 \omega_1^4 - \omega_1^2 \omega_2^4$$

$$\omega_0^4 (\omega_1^2 - \omega_2^2) = \omega_2^2 \omega_1^2 (\omega_1^2 - \omega_2^2)$$

$$\omega_0^4 = \omega_1^2 \omega_2^2$$

$$\omega_0^2 = \omega_1 \omega_2 \quad \text{Q.E.D.}$$

(b) For Fig. 16.4:

$$\omega_{p1} = 8100 \text{ rad/s}$$

$$\omega_{p2} = 10000 \text{ rad/s}$$

$$A_{\text{max}} = 1 \text{ dB}$$

$$\omega_0^2 = (8100)(10000)$$

$$\omega_0 = 9000 \text{ rad/s}$$

$$|T(j\omega_{p1})| = |T(j\omega_{p2})| = 10^{-1/20}$$

$$= 0.8913$$

$$|T(j\omega_0)| = \frac{\omega_0 a_1}{\sqrt{(\omega_0^2 - \omega_0^2)^2 + \left(\frac{\omega_0^2}{Q}\right)^2}} = 1$$

$$\Rightarrow \frac{\omega_0 a_1}{\omega_0^2 / Q} = 1$$

$$\therefore \frac{Q a_1}{\omega_0} = 1 \Rightarrow a_1 = \frac{\omega_0}{Q}$$

$$|T(j\omega_{p1})|^2 = |T(j0.9\omega_0)|^2 = 0.8913^2$$

$$\frac{(\omega_o/Q)^2(0.9\omega_o)^2}{(\omega_o^2 - (0.9\omega_o)^2)^2 + \left(\frac{0.9\omega_o}{Q}\right)^2} = 0.8913^2$$

$$\left(\frac{\omega_o}{Q}(0.9\omega_o)\right)^2 = 0.8913^2 \left[(\omega_o^2 - (0.9\omega_o)^2)^2 + \left(\frac{0.9\omega_o}{Q}\right)^2 \right]$$

$$\frac{0.81\omega_o^4}{Q^2} = 0.8913^2 \left[W_o^4(1 - 0.81)^2 + \frac{0.81\omega_o^4}{Q^2} \right]$$

$$\frac{0.81\omega_o^4}{Q^2}(1 - 0.8913^2) = 0.8913^2\omega_o^4 \times (1 - 0.81)^2$$

SUB $\omega_o = 9000$ gives

$$Q = 2.41$$

$$\text{Now } Q_1 = \frac{\omega_o}{Q} = 0.415\omega_o$$

$$\therefore T(S) = \frac{0.415\omega_o S}{S^2 + 0.415\omega_o S + \omega_o^2}$$

IF $WS_1 = 3000$ rad/s

$$|T(j3000)| = \frac{0.415\omega_o(3000)}{\sqrt{(W_o^2 - 3000^2)^2 + (\omega_o 3000 \times .415)^2}}$$

$$= 0.1537$$

$$\therefore A_{\min} = -20\log(0.1537)$$

$$= 16.3\text{dB}$$

Now ω_{s1} and ω_{s2} are geometrically symmetrical about ω_o :

$$\omega_{s1}\omega_{s2} = \omega_o^2$$

$$\omega_{s2} = \frac{9000^2}{3000}$$

$$= 27000 \text{ rad/s}$$

16.30

From exercise 16.15

$$Q = \frac{\omega_o}{BW\sqrt{10^{A/10} - 1}}$$

$$\omega_o = 2\pi(60) BW = 2\pi 6 A = 20\text{dB} \\ = 1.005$$

$$T(S) = a_2 \frac{S^2 + \omega_o^2}{S^2 + S\frac{\omega_o}{Q} + \omega_o^2}$$

$$|T(0)| = \frac{a_2\omega_o^2}{\omega_o^2} = 1 \leftarrow \text{DC Gain}$$

$$Q_2 = 1$$

$$T(S) = \frac{S^2 + (2\pi 60)^2}{S^2 + S\frac{2\pi 60}{1.005} + (2\pi 60)^2}$$

$$T(S) = \frac{S + 1.421 \times 10^5}{S^2 + 375.1s + 1.421 \times 10^5}$$

16.31

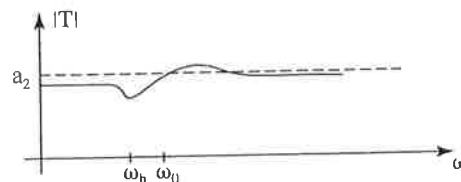
FOR ALL PASS:

$$T(S) = Q_2 \frac{S^2 - SW_o/Q + \omega_o^2}{S^2 + SW_o/Q + \omega_o^2}$$

If Zero frequency < pole frequency

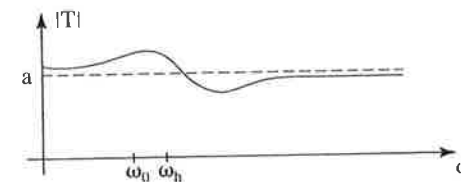
$$T(S) = Q_2 \frac{S^2 - SW_n/Q + \omega_n^2}{S^2 + SW_o/Q + \omega_o^2} \quad \omega_n < \omega_o$$

$$\text{At DC: } |T| = a \frac{\omega_n^2}{\omega_o^2} \quad \text{where } \frac{\omega_n^2}{\omega_o^2} < 1$$



If Zero frequency > pole frequency then $\omega_n > \omega_o$

$$\text{At DC: } |T| = Q_2 \frac{\omega_n^2}{\omega_o^2} \quad \text{where } \frac{\omega_n^2}{\omega_o^2} > 1$$



16.32

$$T(S) = \frac{S^2 - SW_o/Q_1 + \omega_o^2}{S^2 + SW_o/Q_2 + \omega_o^2} a$$

Zero $Q <$ pole $Q \Rightarrow Q_1 < Q_2$

At $W = W_o$:

$$|T| = \frac{a_2\omega_o^2/Q_1}{\omega_o^2/Q_2} = \frac{a_2Q_2}{Q_1} > a_2$$

If $Q_1 > Q_2$

$$|T(j\omega_o)| = \frac{Q_2Q_o}{Q_1} < Q_2$$