WEEK 2

2.28

From example 2.2

$$\frac{v_o}{v_I} = -\frac{R_2}{R_1}\left(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right)$$

Here $R_1 = R_2 = R_4 = 1\ \text{M}\Omega$

$$\therefore\ \frac{v_o}{v_I} = -\left(1 + 1 + \frac{1}{R_3}\right) = -\left(2 + \frac{1\ \text{M}}{R_3}\right)$$

$$\frac{v_o}{v_I} + 2 = -\frac{1\ \text{M}\Omega}{R_3}$$

$$R_3 = -\frac{1\ \text{M}}{\left(\frac{v_o}{v_I} + 2\right)}$$

a. $\frac{v_o}{v_I} = -200\ \frac{\text{V}}{\text{V}}$

$$R_3 = -\frac{1\ \text{M}}{(-200 + 2)} = 5.05\ \text{k}\Omega$$

b. $\frac{v_o}{v_I} = -20\ \frac{\text{V}}{\text{V}}$

$$R_3 = -\frac{1\ \text{M}}{(-20 + 2)} = 55.6\ \text{k}\Omega$$

c. $\frac{v_o}{v_I} = -2\frac{\text{V}}{\text{V}}\quad R_3 = -\frac{1\ \text{M}\Omega}{(-2 + 2)} = \infty$

2.29

$R_2 / R_1 = 1000$, $R_2 = 100\ \text{k}\Omega \Rightarrow R_1 = 1000\ \Omega$

a) $R_{in} = R_1 = 100\ \Omega$

b) $\frac{v_o}{v_I} = \frac{-R_2}{R_1}\left(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right) = -1000$

If $R_2 = R_1 = R_4 = 100\ \text{K} \Rightarrow R_3 = \frac{100\ \text{K}}{1000 - 2} \simeq$

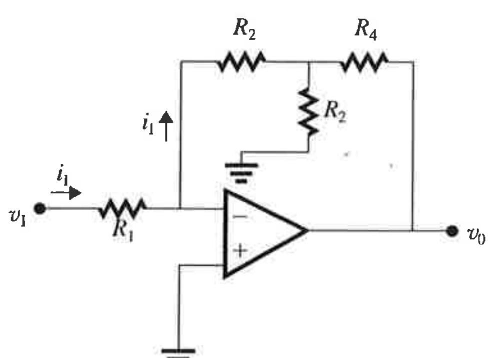
$100\ \Omega$

$R_{in} = R_1 = 100\ \text{k}\Omega$

2.30

$$i_I = \frac{v_I}{R_1},\quad v_x = -i_I R_2 = -\frac{v_I}{R_1} R_2$$

So $\frac{v_x}{v_I} = -\frac{R_2}{R_1}$ or $\frac{v_I}{v_x} = -\frac{R_1}{R_2}$



$$v_x = v_o \frac{(R_2 \parallel R_3)}{(R_2 \parallel R_3) + R_4}$$

$$= v_o \frac{R_2 R_3}{R_2 R_3 + R_2 R_4 + R_3 R_4}$$

$$\frac{v_o}{v_x} = \frac{R_2 R_3 + R_2 R_4 + R_3 R_4}{R_2 R_3}$$

$$= 1 + \frac{R_4}{R_3} + \frac{R_4}{R_2}$$

$$\frac{v_o}{v_I} = \frac{v_o / v_x}{v_I / v_x} = \frac{1 + \frac{R_4}{R_3} + \frac{R_4}{R_2}}{-\frac{R_1}{R_2}}$$

$$= -\frac{R_2}{R_1}\left(1 + \frac{R_4}{R_3} + \frac{R_4}{R_2}\right)$$

2.31

a) $R_1 = R$

$$R_2 = (R \parallel R) + \frac{R}{2} = \frac{R}{2} + \frac{R}{2} = R$$

$$R_3 = (R_2 \parallel R) + \frac{R}{2} = (R \parallel R) + \frac{R}{2} = R$$

$$R_4 = (R_3 \parallel R) + \frac{R}{2} = (R \parallel R) + \frac{R}{2} = R$$

b) $v = RI = RI_1 \Rightarrow I_1 = I$

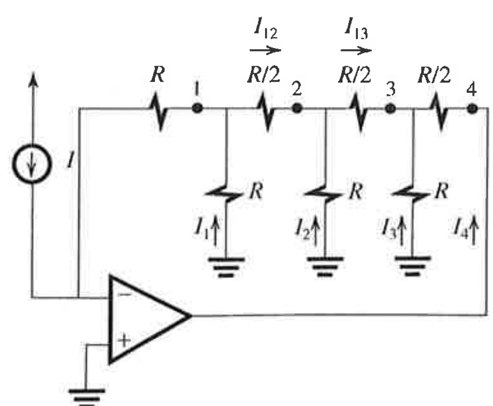
$$I_{12} = I + I = 2I \Rightarrow v_1 + 2I \times \frac{R}{2} = RI_2$$

$$RI + RI = RI_2 \Rightarrow I_2 = 2I$$

$$I_{13} = I_2 + I_{12} = 4I \Rightarrow v_2 + 4I \times \frac{R}{2} = RI_3$$

$$R \times 2I + 4I \times \frac{R}{2} = RI_3 \Rightarrow I_3 = 4I,$$

$$I_4 = -(4I + 4I)\ I_4 = 8I$$



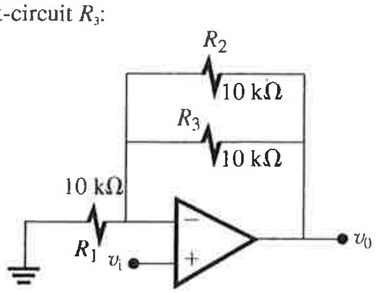
c) $v_1 = I_1 R = -IR$

$v_2 = -I_2 R = -2IR$

$v_3 = -I_3 R = -4IR$

$$v_4 = -I_3 R + I_4 \frac{R}{2} = -4IR - 8I\frac{R}{2} = -8IR$$

Short-circuit R_3:



$$\frac{v_o}{v_i} = 1$$

2.46

$v_+ = v_- = v_o = R \times i,\ i = 100\ \mu A$ when

$v = 10V$

$$\Rightarrow R = \frac{10}{0.1\ mA} = 100\ k\Omega$$

As indicated, i only depends on R and v and the meter resistance does not affect i.

2.47

Refer to the circuit in P 2.47:

a) Using superposition, we first set

$v_{P_1} = v_{P_2} \ldots = o$. The output voltage that

results in response to $v_{N1}, v_{N2}, \ldots v_{Nn}$ is:

$$v_{ON} = -\left[\frac{R_F}{R_{N1}}v_{N1} + \frac{R_F}{R_{N2}}v_{N2} + \ldots + \frac{R_F}{R_{Nn}}v_{Nn}\right]$$

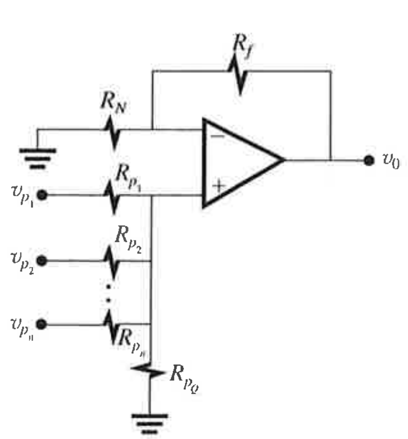
Then we set $v_{N1} = v_{N2} = \ldots = o$, then:

$R_N = R_{N1} \parallel R_{N2} \parallel R_{N3} \parallel \ldots \parallel R_{Nn}$

The circuit simplifies to:

$$v_{op} = \left(1 + \frac{R_F}{R_N}\right)\times$$

$$\left(v_{P1}\frac{1/R_{P1}}{\frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{Pn}}} + v_{P2}\frac{1/R_{P2}}{\frac{1}{R_{P1}} + \ldots + \frac{1}{R_{Pn}}}\right.$$



$$\left.\ldots v_{Pn}\frac{1/R_{Pn}}{1/R_{P1} + \ldots + \frac{1}{R_{Pn}}}\right)$$

$$v_{op} = \left(1 + \frac{R_F}{R_N}\right)\left(v_{P1}\frac{R_P}{R_{P1}} + v_{P2}\frac{R_P}{R_{P2}} + \ldots + \frac{R_P}{R_{Pn}}v_{Pn}\right)$$

where:

$R_P = R_{P1} \parallel R_{P2} \parallel \ldots \parallel R_{Pn}$

when all inputs are present:

$v_o = v_{ON} + v_{OP}$

$$= -\left(\frac{R_F}{R_{N1}}v_{N1} + \frac{R_F}{R_{N2}}v_{N2} + \ldots\right) +$$

$$\left(1 + \frac{R_F}{R_N}\right)\left(\frac{R_P}{R_{P1}}v_{N1} + \frac{R_P}{R_{P2}}v_{N2} + \ldots\right)$$

b) $v_o = -2v_{N1} + v_{P1} + 2v_{P2}$

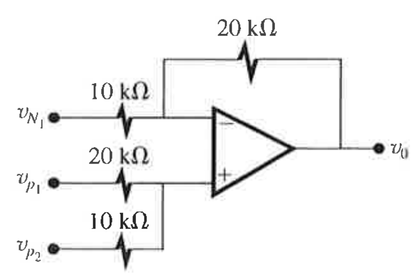
$$\frac{R_F}{R_{N1}} = 2,\ R_{N1} = 10\ k\Omega \Rightarrow R_F = 20\ k\Omega$$

$$\left(1 + \frac{R_F}{R_N}\right)\left(\frac{R_P}{R_{P1}}\right) = 1 \Rightarrow 3\frac{R_P}{R_{P1}} = 1 \Rightarrow R_{P2} = \frac{R_{P1}}{2}$$

$$\left(1 + \frac{R_F}{R_N}\right)\left(\frac{R_P}{R_{P2}}\right) = 2 \Rightarrow 3\frac{R_P}{R_{P2}} = 2 \Rightarrow R_{P2} = \frac{R_{P1}}{2}$$

where $R_P = \dfrac{R_{P1} \times R_{P2}}{R_{P1} + R_{P2}}$ (ignoring R_{Po})

Note that if the results from the last 2 constraints differ, we would use an additional resistor connected from the positive input to ground. (R_{Po})



2.48

$v_o = v_{I1} + 3v_{I2} - 2(v_{I3} + 3v_{I4})$

Refer to P2.47

$$\frac{R_F}{R_{N3}} = 2 \text{ if } R_{N3} = 10\ k\Omega \Rightarrow R_F = 20\ k\Omega$$

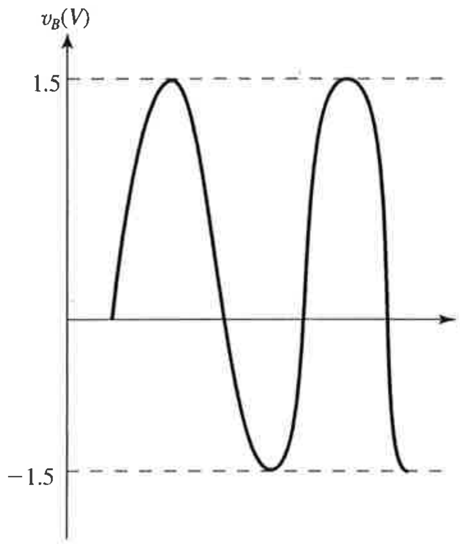
$$\frac{R_F}{R_{N4}} = 6 \Rightarrow R_{N4} = \frac{20}{6} = 3.3\ k\Omega$$

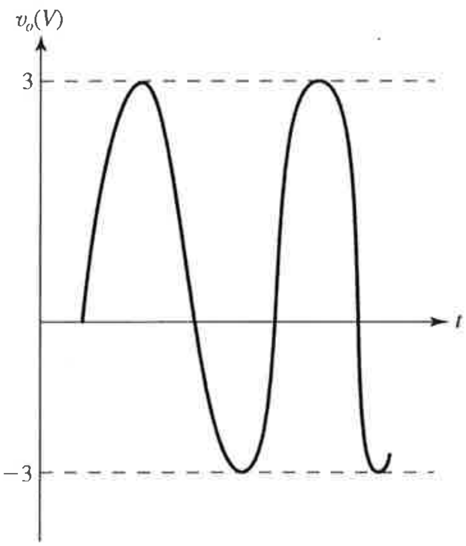
$$R_N = R_{N3} \parallel R_{N4} = 10\ K \parallel 3.3\ k = 2.48\ k\Omega$$

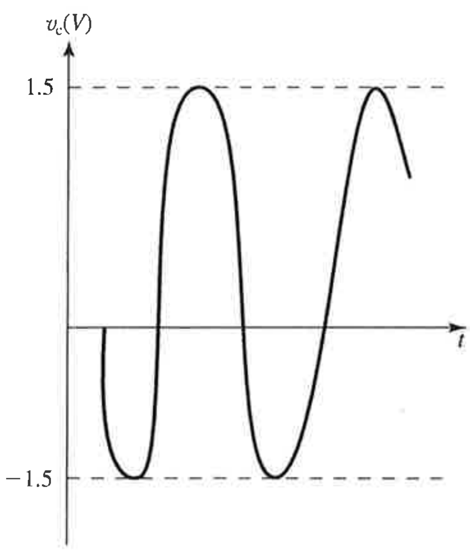
$$\left(1 + \frac{R_F}{R_N}\right)\frac{R_P}{R_o} = 1 \Rightarrow \left(1 + \frac{20}{2.48}\right)\frac{R_P}{R_{P1}}$$

$$= 1 \Rightarrow 9.06\ R_P = R_{P1}$$

$R_P = R_{P1} \parallel R_{P2} \parallel R_{P3} \Rightarrow R_P$





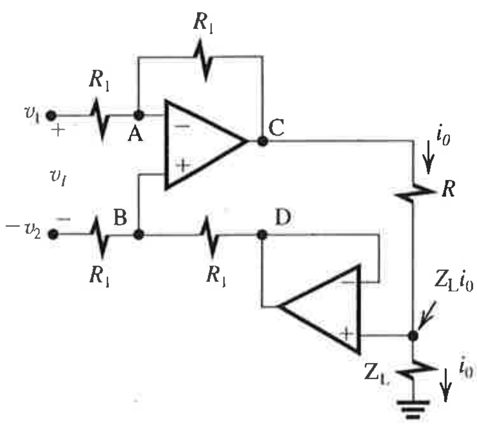


c) v_B and v_C can be ± 14 V or 28 V P-P.

$-28 \le v_o \le 28$ or 56 VP-P.

$$v_{orms} = 19.8 \text{ V} = \frac{28}{\sqrt{2}}$$

2.78



Refer to Fig. P.2.78.a:

Since the inputs of the op-amp do not draw any current, v_I appears across R_1

$$i_o = \frac{v_1}{R}$$

Fig. P2.78.B

$$v_D = Z_L i_o$$

we use superposition:

$v_1 = v_1 - v_2$

See next page.

$$v_1 \text{ only: } V_B = \frac{V_O}{2} = \frac{z_L i_{o1}}{2}$$

$$\frac{v_1 - \frac{z_L i_{o1}}{2}}{R_1} = \frac{\frac{z_L i_{o1}}{2} - i_{o1}(z_L + R)}{R_1}$$

$$\Rightarrow v_1 = i_{o1}R \Rightarrow i_{o1} = \frac{v_1}{R}$$

Now if only $(-v_2)$ is applied:

$$v_B = \frac{-v_2 + z_L i_{o2}}{2}, \; v_A = \frac{i_{o2} \times (R + z_L)}{2}$$

$$v_A = v_B \Rightarrow -v_2 + z_L i_{o2} = i_{o2}R + i_{o2}z_L$$

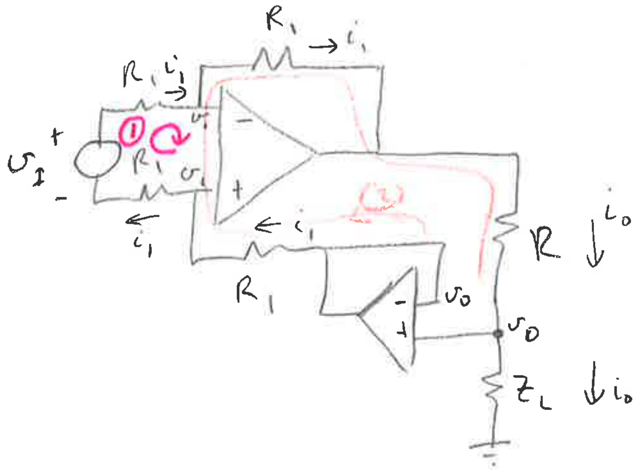
$$-v_2 = i_{o2}R \Rightarrow i_{o2} = \frac{-V_2}{R}$$

The total current due to both sources is:

$$i_o = i_{o1} + i_{o2} = \frac{v_1}{R} - \frac{v_2}{R} = \frac{v_I}{R}$$

The circuit in Figure P2.78(a) has ideally infinite input resistance, and if requires that both terminals of Z_L be available, while the other circuit has finite input resistance with one side of Z_L grounded.

2.78 b)



Equal currents + voltages are marke

KVL around loop 1 $\quad -i_1R_1 + v_I - i_1R_1 = 0 \quad$ so $\quad i_1 = \frac{v_I}{2R_1}$

KVL around loop 2 $\quad -i_1R_1 - i_1R_1 - i_oR = 0$

$$-\frac{v_I}{2R_1}R_1 - \frac{v_I}{2R_1}R_1 - i_oR = 0$$

so $\quad i_o = -\frac{v_I}{R}$

2.91

$RC = 10^{-3}\text{s}$ when

$C = 10\text{mF} \Rightarrow R = 100\ \text{k}\Omega$

$\frac{v_O}{v_i} = -\text{sRC},\ \frac{v_O}{v_i}(jw) = -jwRC$

$\varphi = -90°$ always

$\left|\frac{v_O}{v_i}\right| = 1 \Rightarrow w = \frac{1}{\text{unity } RC} = 1\text{krad/s}$

Gain is 10 times the unity gain, when = the frequency is 10 times the unity gain frequency. Similarly for $w = \frac{1}{10}$ krad/s, gain is 0.1 V/V. (for $w = 10$ krad/s, gain = 10 V/V)

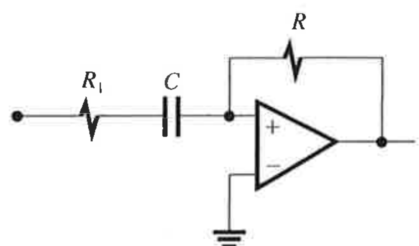
for high frequency C is short circuited,

$\frac{v_O}{v_i} = \frac{-R}{R_1} = -100 \Rightarrow R_1 = 1\ \text{k}\Omega$

$\frac{v_O}{v_i} = \frac{-RCs}{R_1Cs + 1} = \frac{-10^{-3}s}{10^{-5}s + 1}$

$\Rightarrow w_{3\text{db}} = 100\ \text{krad/s}$ or

$f_{3db} = 15.9$ kHz



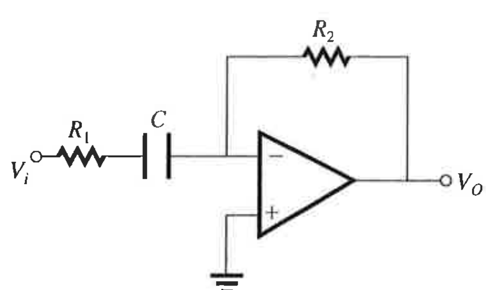
for unity gain:

$|10^{-3}s| = |10^{-5}s + 1| \Rightarrow w_H = 1.01$ krad/s

if $w = 10.1\text{krad/s}$: $\left|\frac{v_O}{v_i}\right| = \frac{10.1}{1.01} = 10,$

$\varphi = -95.77°$

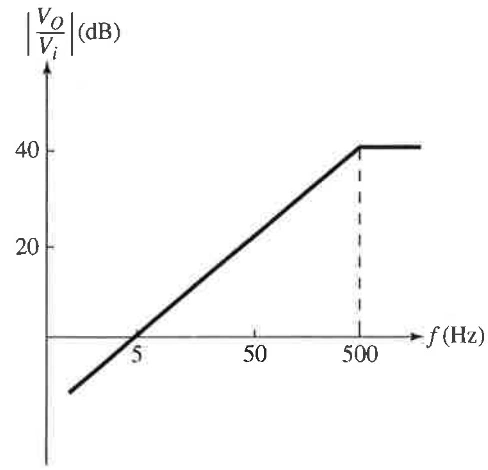
2.92



Let $Z_1 = R_1 + \frac{1}{sC},\ Z_2 = R_2$

Gain $= \frac{v_O}{v_i} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1 + \frac{1}{sC}}$

$= \frac{-(R_2/R_1)s}{s + \frac{1}{R_1C}}$



This is the transfer function of a single time constant high pass filter having High frequency gain $= (R_2/R_1)$ and 3dB frequency at $w_O = \frac{1}{R_1C}$.

At high frequency the input impedance abbroaches R_1 as 20 $\frac{1}{jwC}$ becomes very small

Select $R_1 = 10\ \text{k}\Omega$

Gain = 40 dB = 100 V/V

For a gain of 100, $\frac{R_2}{R_1} = 100$

and $R_1 = 10\ \text{k}\Omega$

$\therefore R_2 = 10\ \text{k}\Omega \times 100$

$= 1\ \text{M}\Omega$

For a 3dB frequency of 500 Hz

$\frac{1}{R_1C} = 2\pi \times 500$

$C = \frac{1}{2\pi \times 500 \times R_1} = 32\ \text{nF}$

From the Bode-plot the gain $\frac{v_O}{v_i}$ reduces to unity at 5 Hz

2.93

Refer to the circuit in fig P 2.126:

$\frac{v_O}{v_i} = -\frac{Z_2}{Z_1} = -\frac{1}{Z_1Y_2} = -\frac{1}{\left(R_1 + \frac{1}{sC_1}\right)\left(\frac{1}{R_2} + sC_2\right)}$

$\frac{v_O}{v_i} = -\frac{R_2/R_1}{\left(1 + \frac{1}{R_1C_1s}\right)(1 + sR_2C_2)}$

$$\frac{v_O}{v_i}(jw) = \frac{-R_2/R_1}{\left(1 + \frac{1}{jwR_1C_1}\right)(1 + jwR_2C_2)}$$

$$= \frac{-R_2/R_1}{\left(1 + \frac{w_1}{jw}\right)\left(1 + j\frac{w}{w_2}\right)}$$

where $w_1 = \frac{1}{R_1C_1}$, $w_2 = \frac{1}{R_2C_2}$

a) for $w = w_1 << w_2$

$$\frac{v_O}{v_i}(jw) \simeq \frac{-R_2/R_1}{\left(1 + \frac{w}{jw}\right)} \simeq \frac{-R_2R_1}{w_1/jw} \simeq -j\frac{R_2}{R_1}\frac{w}{w_1}$$

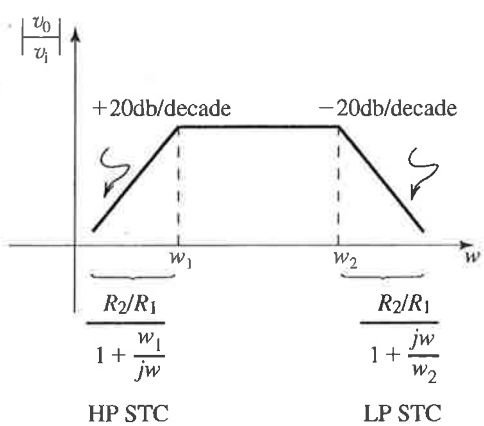
b) for $w_1 << w << w_2$

$$\frac{v_O}{v_i}(jw) \simeq -\frac{R_2}{R_1}$$

c) for $w >> w_2$ and $w_2 >> w_1$:

$$\frac{v_O}{v_i}(jw) \simeq \frac{-R_2R_1}{1 + jw/w_2} \simeq \frac{-R_2R_1}{jw/w_2} = j\left(\frac{R_2}{R_1}\right)\left(\frac{w_2}{w}\right)$$

from the results of a), b) and c) we can draw the Bode-plot:



Design: $\frac{R_2}{R_1} = 1000$ (60 dB gain in the mid-frequency range)

R_{in} for $w >> w_1$

$= R_1 = 1\text{ k}\Omega \Rightarrow R_2 = 1\text{ M}\Omega$

$f_1 = 100\text{ Hz} \Rightarrow w_1 = 2\pi \times 100 = \frac{1}{R_1C_1}$

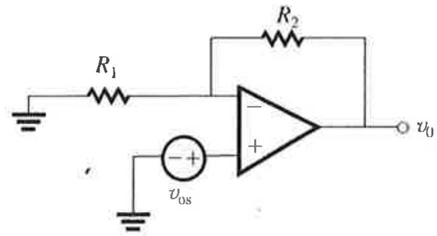
$\Rightarrow C_1 = 1.59\ \mu\text{F}$

$f_2 = 10\text{ Hz} \Rightarrow w_2 = 2\pi \times 10 \times 10^3 = \frac{1}{R_1C_1}$

$\Rightarrow C_2 = 15.9\text{ pF}$

2.94

Inverting configuration



$v_O = v_{OS}\left(1 + \frac{R_2}{R_1}\right)$

$-0.4 = v_{OS}\left(1 + \frac{100}{1}\right)$

$v_{OS} \simeq 4\text{ mV}$

2.95

$v_{OS} = \pm 2\text{ mV}$

$v_O = 0.01\sin\omega t \times 200 + v_{OS} \times 200$

$= 2\sin\omega t \pm 0.4\text{ V}$

2.96

Input offset voltage = 5 mV

Output dc offset voltage =

5 mV × closed loop gain

= 5 mV × 1000

= 5V

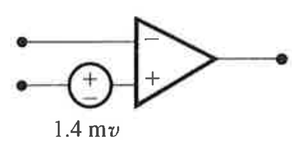
The maximum amplifier of an input sinusoid that results in an output peak amplifier of 13 – 5 = 8V is given by:

$v_i = \frac{8}{1000} = 8\text{ mV}$

If amplifier is capacitively coupled then

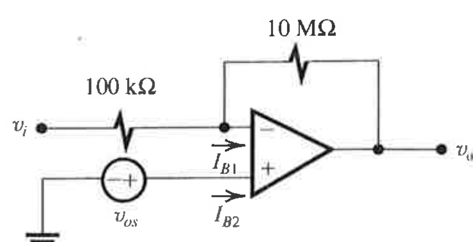
$v_{i\,max} = \frac{13}{1000} = 13\text{ mV}$

2.97

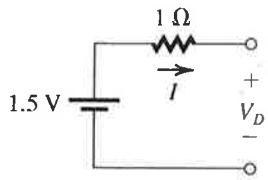


$v_{OS} = \frac{1.4}{100} = 1.4\text{ mV}$

2.98



4.1



The diode can be reverse-blased and thus no current would flow, or forward-blased where current would flow.
(a) Reverse biased I = OA $V_D = 1.5$ V
(b) Foward biased $I = 1.5$ A $V_D = 0$ V

4.2
(a) Diode is conducting and thus has a OV drop across it. Consequently
$V = -5$V

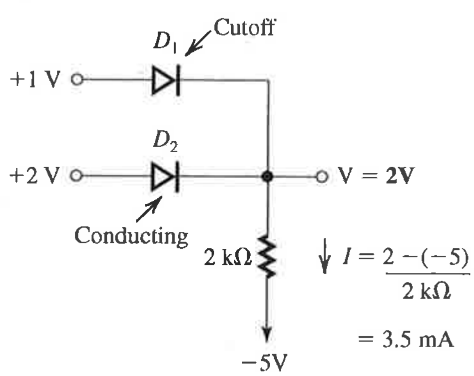
$$I = \frac{5-(-5)}{10\ \text{k}\Omega} = 0.1\ \text{mA}$$

(b) Diode is cut off.
$V = 5$V $I =$ OA
(c) Diode is conducting
$V = 5$V

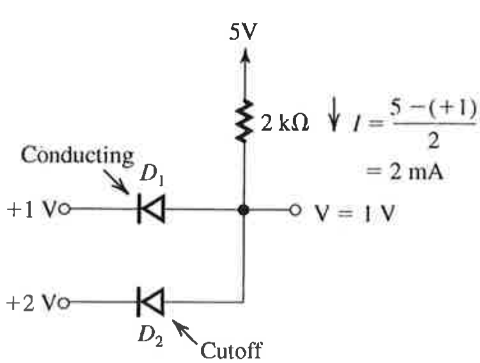
$$I = \frac{5-(-5)}{10\ \text{k}\Omega} = 0.1\ \text{mA}$$

(d) Diode is cut off.
$V = -5$ V $I =$ OA

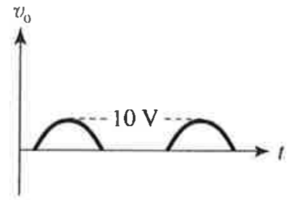
4.3
(a)



(b)

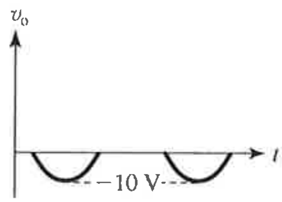


4.4
(a)



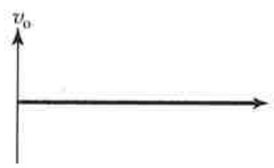
$V_{p+} = 10$ V $V_{p-} = 0$ V
$f = 1$ kH$_3$

(b)



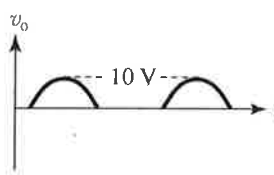
$V_{p+} = 0$ V $V_{p-} = -10$ V
$f = 1$ kH$_3$

(c)



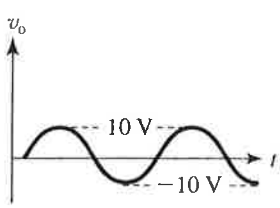
$v_O = 0$ V
Neither D_1 nor D_2 conducts so there is no output.

(d)



$V_{p+} = 10$ V $V_{p-} = 0$ V $f = 1$ kH$_3$
Both D_1 and D_2 conduct when $V_I > 0$

(e)



$$i_{B(avg)} = \frac{\pi - 2\arcsin(3/10)}{2\pi} \cdot 60 \text{ mA} = 24.2 \text{ mA}$$

For peak V_i reduced by 10%

$$i_{B(peak)} = 60 \text{ mA}$$

$$i_{B(avg)} = \frac{\pi - 2\arcsin(3/9)}{2\pi} \cdot 60 \text{ mA} = 23.5 \text{ mA}$$

4.6

A	B	x	y
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

$x = A\,B \qquad y = A + B$

x and y are the same for

$A = B$

x and y are opposite if $A \neq B$

4.7

The case for the highest current in a single diode is when only one input is high:

$V_y = 5$ V

$$\frac{V_y}{R} \leq 0.2 \text{ mA} \Rightarrow R \geq 25 \text{ k}\Omega$$

4.8

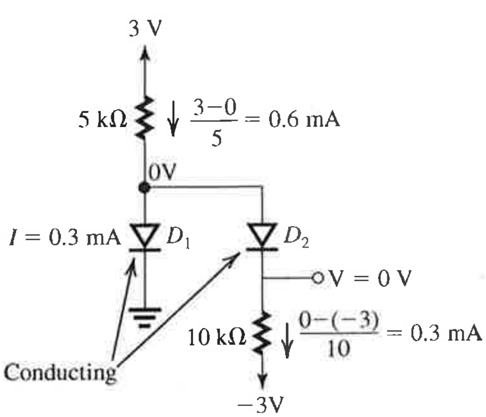
The maximum input current occurs when one input is low and the other two are high.

$$\frac{5-0}{R} \leq 0.1 \text{ mA}$$

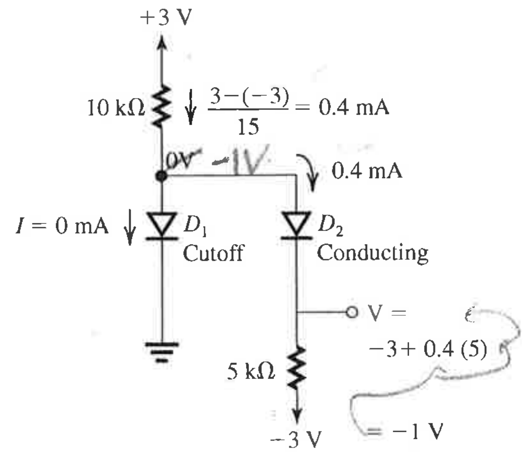
$R \geq 50$ kΩ

4.9

(a)

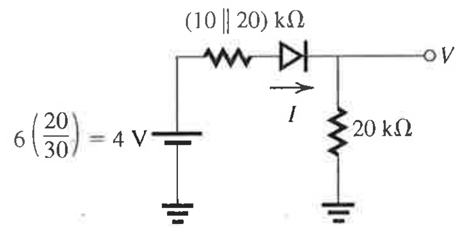


(b)



4.10

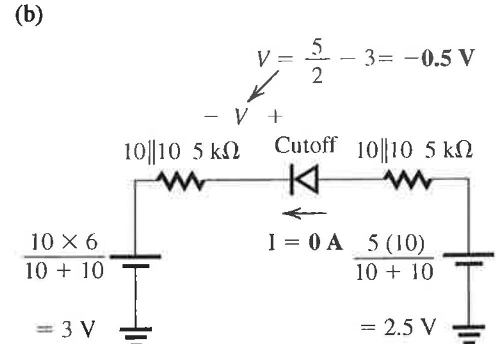
(a)



$$I = \frac{4}{(10\,\|\,20) + 20} = 0.15 \text{ mA}$$

$$V = \frac{20}{(10\,\|\,20) + 20} \times 4 = 3 \text{ V}$$

(b)

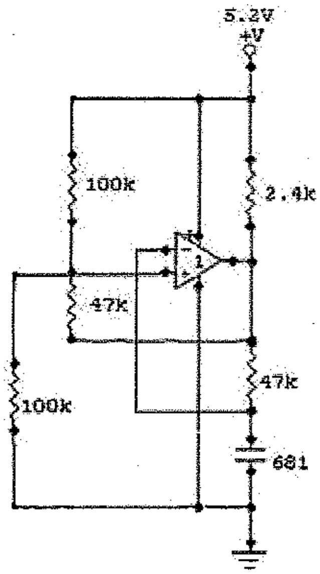


4.11

$$R \geq \frac{120\sqrt{2}}{50} \geq 3.4 \text{ k}\Omega$$

The largest reverse voltage appearing across the diode is equal to the peak input voltage

$120\sqrt{2} = 169.7$ V



This is relaxation oscillator, similar to lab

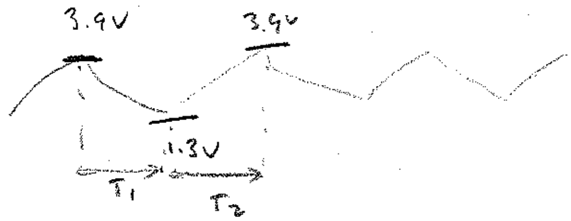
approx 47k ≈ 50k
47+2.4k ≈ 50k

$$v_{+,HI} = 5.2 \cdot \frac{100}{100 + 100\|50} = 3.9V$$

$$v_{+,LO} = 5.2 \cdot \frac{100\|50}{100\|50 + 100} = 1.3$$

let $\tau = 680pF \cdot 50k \approx .034\,mSec$

$v_- = v_c =$



$$v(t) = v(\infty) + (v(0) - v(\infty))e^{-t/\tau}$$

$$\frac{v(t) - v(\infty)}{v(0) - v(\infty)} = e^{-t/\tau}$$

$$t = \tau \ln\left(\frac{v(0) - v(\infty)}{v(t) - v(\infty)}\right)$$

$$T_1 = .034mS \cdot \ln\left(\frac{3.9-0}{1.3-0}\right) = .037mS$$

$$T_2 = .034 \cdot \ln\left(\frac{1.3-5}{3.9-5}\right) = .041mS$$

$$f_{req} = \frac{1}{.037+.041} = 12.8kHz$$

So... we get approx a 12.8 kHz triangle wave generator from 1.3 → 3.9 V