WEEK 11

1)

```
>> [n,w0]=buttord(2*pi*3000, 2*pi*8000, 3, 50, 's')
n =
     6
w0 =
   1.9258e+04
>> [b,a]=butter(6,1,'s')
b =
         0         0         0         0         0         0    1.0000
a =
    1.0000    3.8637    7.4641    9.1416    7.4641    3.8637    1.0000
>> [sos,g]=tf2sos(b,a)
sos =
         0         0    1.0000    1.0000    1.9319    1.0000
         0         0    1.0000    1.0000    0.5176    1.0000
         0         0    1.0000    1.0000    1.4142    1.0000
g =
    1.0000
>> q=1./sos(:,5)
q =
    0.5176
    1.9319
    0.7071
>>
```

$\Leftarrow$ 6th order filter

Stage

num

den

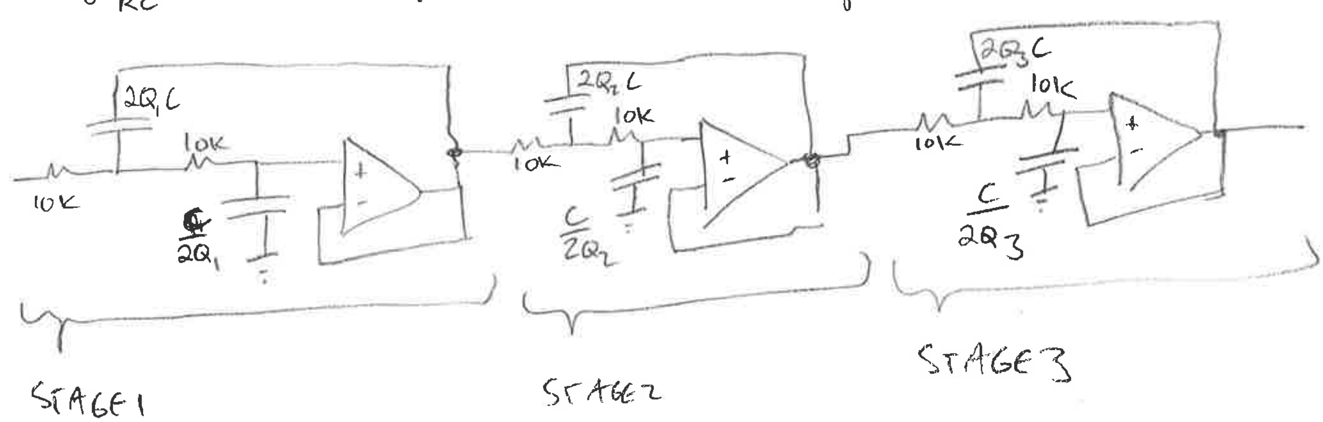
$0.5176 = Q_1$

$1.9319 = Q_2$

$0.7071 = Q_3$

$$H(s) = \underbrace{\frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q_1}s + \omega_0^2}}_{\text{STAGE 1}} \cdot \underbrace{\frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q_2}s + \omega_0^2}}_{\text{STAGE 2}} \cdot \underbrace{\frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q_3}s + \omega_0^2}}_{\text{STAGE 3}}$$

2) $\omega_0 = \frac{1}{RC} = 2\pi \cdot 3000$ $\quad R = 10k$ $\quad C = 5.3\,nF = 5300\,pF$



3) $K=1$ $\quad$ Replace R_1 by $\frac{1}{sC_\alpha} = \frac{1}{sC}$ $\quad$ Replace R_2 by $\frac{1}{sC_\beta} = \frac{1}{sC}$ $\quad$ Replace $\frac{1}{sC_1}$ by R_α

$C_1 = \frac{1}{sR_\alpha}$ $\qquad C_2 = \frac{1}{sR_\beta}$

$$H(s) = \frac{s^4 C^2 R_\alpha R_\beta}{s^2 + [s^2 C R_\alpha + s^2 C R_\alpha]s + s^4 C^2 R_\alpha R_\beta} = \frac{s^2}{s^2 + \left[\frac{1}{CR_\beta} + \frac{1}{CR_\beta}\right]s + \frac{1}{R_\alpha R_\beta C^2}}$$

$R_\alpha = \frac{R}{2Q}$

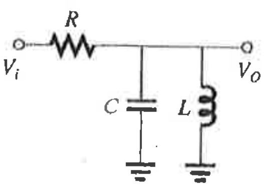
$R_\beta = 2RQ$

$$H(s) = \frac{s^2}{s^2 + \left[\frac{\omega_0}{Q}\right]s + \omega_0^2}$$

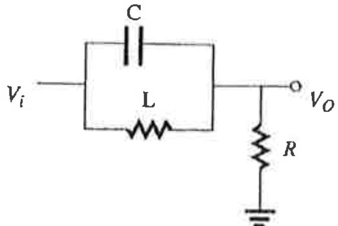
4) $R_1 = 680\Omega$ $R_2 = 2K + 5k = 7k$ $C = .01\mu F$

$f = \frac{1.44}{(R_1 + 2R_2)C} = 9.8\,kHz$ → (very close to the 10kHz Band pass center frequency.

$D.C. = \frac{0.69(R_1+R_2)\cancel{C}}{0.69(R_1+2R_2)\cancel{C}} = 0.52$ → close to square wave.



⇒ INTERCHANGE V_i & gnd to get:



5)

16.62

for fig 16.18(d)

$$T(S) = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

$$\omega_O = \frac{1}{\sqrt{LC}} \quad Q = R\sqrt{\frac{C}{L}}$$

For w_o

$$\frac{\partial \omega_O}{\partial L} = \frac{\partial (LC)^{-1/2}}{\partial L} = -\frac{1}{2}L^{-3/2}C^{-1/2} = \frac{\omega_O}{2L}$$

$$\frac{\partial \omega_O}{\partial C} = -\frac{\omega_O}{2C}$$

$$\frac{\partial \omega_O}{\partial R} = 0$$

$$\therefore S_L^{\omega_O} = \frac{\partial \omega_O}{\partial L}\frac{L}{\omega_O} = -1/2$$

$$S_C^{\omega_O} = \frac{\partial \omega_O}{\partial C} \times \frac{C}{\omega_O} = -1/2$$

$$S_R^{\omega_O} = \frac{\partial \omega_O}{\partial R}\frac{R}{\omega_O} = 0$$

For Q

$$\frac{\partial Q}{\partial L} = \frac{R\sqrt{C}}{L\sqrt{L}}\left(\frac{-1}{2}\right) = \frac{-Q}{2L}$$

$$\frac{\partial Q}{\partial C} = \frac{1}{2}\frac{R}{\sqrt{LC}} = \frac{1}{2}\frac{R\sqrt{C}}{C\sqrt{L}} = \frac{Q}{2C}$$

$$\frac{\partial Q}{\partial R} = \sqrt{C/L} = \frac{R}{R}\sqrt{C/L} = Q/R$$

$$S_L^Q = -\frac{Q}{2C} \times \frac{L}{Q} = -\frac{1}{2}$$

$$S_C^Q = \frac{Q}{2C} \times \frac{C}{Q} = \frac{1}{2}$$

$$S_R^Q = -\frac{Q}{R}\cdot\frac{R}{Q} = 1$$

16.63

$y = uv$

$$S_x^y = \frac{\partial (uv)}{\partial x}\frac{x}{uv}$$

$$= V\frac{\partial u}{\partial x}\frac{x}{uv} + u\frac{\partial V}{\partial x}\frac{x}{uv}$$

$$= \frac{\partial u}{\partial x}\frac{x}{u} + \frac{\partial V}{\partial x}\frac{x}{V}$$

$$= S_x^u + S_x^v$$

Part (b) $y = u/v$

$$S_x^y = \frac{\partial y}{\partial x}\cdot\frac{x}{y} + \frac{\partial (u/v)}{\partial x}\frac{xv}{u}$$

$$= \frac{1}{V}\frac{\partial u}{\partial x}\frac{vx}{u} + \frac{-u}{V^2}\frac{\partial V}{\partial x}\cdot\frac{xv}{u}$$

$$= \frac{\partial u}{\partial x}\frac{x}{u} - \frac{\partial V}{\partial x}\frac{x}{v}$$

$$= S_x^u - S_x^v$$

Part (c) $y = ku$

$$S_x^y = \frac{\partial u}{\partial x}\cdot\frac{x}{y} + \frac{\partial (ku)}{\partial x}\frac{x}{ku}$$

$$= k\frac{\partial u}{\partial x}\frac{x}{ku}$$

$$= \frac{\partial u}{\partial x}\frac{x}{u}$$

$$= S_x^u$$

Part (d) $y = u^n$

$$S_x^y = \frac{\partial y}{\partial x}\cdot\frac{x}{y} + \frac{\partial (u^n)}{\partial x}\frac{x}{u^n}$$

$$= nu^{n-1}\frac{\partial u}{\partial x}\cdot\frac{x}{u^n}$$

$$= n\frac{\partial u}{\partial x}\cdot\frac{x}{u^n} = nS_x^u$$

Part (e) $y = f_1(u)$ $u = f_2(x)$

$$S_x^y = \frac{\partial y}{\partial x}\frac{x}{y} \times \frac{\partial f_1(u)}{\partial x}\frac{x}{f_1(u)}$$

$$= \frac{\partial f_1(u)}{\partial x}\frac{\partial u}{\partial x}\cdot\frac{x}{f_1(u)}$$

$$= \frac{\partial f_1}{\partial u}\cdot\frac{\partial f_2}{\partial x}\cdot\frac{x}{f_1}\cdot\frac{u}{u}$$

But $u = f_2$

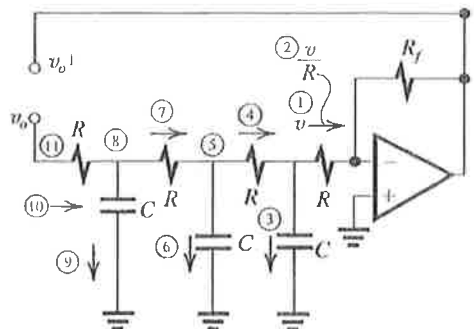
$$\therefore S_x^y = \frac{\partial f_1}{\partial u}\cdot\frac{\partial f_2}{\partial x}\cdot\frac{x}{f_1}\cdot\frac{u}{f_2}$$

$$= \frac{\partial f_1}{\partial u}\cdot\frac{u}{f_1}\cdot\frac{\partial f_2}{\partial x}\frac{x}{f_2}$$

$$= S_u^{f_1}\cdot S_x^{f_2}$$

6)

17.18



(3) $i = scv$

(4) $scv = v/R$

(5) $v + \left(SCv + \frac{v}{R}\right)R = 2v + SCRv$

(6) $2SCv + S^2C^2Rv$

(7) = (6) + (4) = $3SCv + S^2C^2Rv + \frac{v}{R}$

(8) $2v + SCRv + v + 3SCRv + S^2C^2R^2v$

$= 3v + 4SCRv + S^2C^2R^2v$

(9) $3SCv + 4S^2C^2Rv + S^3C^3R^2v$

(10) = (7) + (9)

$= 6SCv + 5S^2C^2Rv + \frac{v}{R} + S^3C^3R^2v$

(11) = (8) + (10) $\times R$

$v_o = 4v + 10SCRv + 6S^2C^2R^2v + s^3c^3R^3v$

$$L(s) = \frac{v_o^1}{v_o} = \frac{vR_f/R}{v(S^3C^3R^3 + 6S^2C^2R^2 + 10SCR + 4)}$$

$$= \frac{R_f/R}{s^3C^3R^3 + 6S^2C^2R^2 + 10SCR + 4}$$

$$L(j\omega) = \frac{R_f/R}{(4 - 6\omega^2C^2R^2) + j(10\omega CR + (\omega^3C^3R^3)}$$

$L(j\omega)$ is purely real if

$$10\omega_O CR = \omega_O{}^3C^3R^3$$

$$\omega_O = \frac{1}{\sqrt{10}}\frac{1}{RC}$$

Given $R = 10\ \text{k}\Omega$, fo = 10 kHz

$$C = \frac{1}{\sqrt{10} \times 10^4 \times 2\pi 10^4}$$

$= 0.503$ nF

Now,

$$|L(j\omega_O)| = \frac{R_f/R}{4 - 6\omega_O^2R^2C^2} \quad \text{sub for } \omega_o$$

$$= \frac{R_f/R}{4 - 6\frac{1}{10R^2C^2}R^2C^2}$$

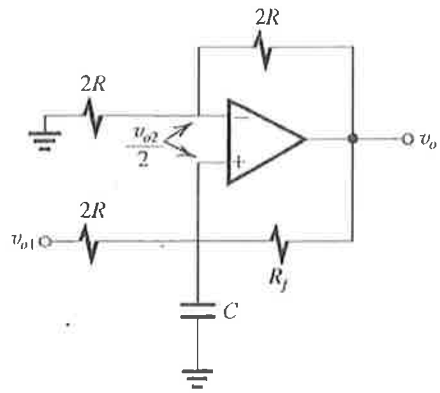
$$= \frac{R_f/R}{4 - 6/10} \geq 1$$

$\therefore R_f/R \geq 3.4$

$R_f \geq 34\ \text{k}\Omega$

17.19

for 2nd indicator



From the voltage divider around the upper branch:

$v+ = v- = \frac{1}{2}v_{02}$

$\Sigma I = 0$ at the input

$$\frac{\frac{1}{2}v_{02} - v_{01}}{2R} + SC\frac{v_{02}}{2} + \frac{\frac{v_{02}}{2} - v_{02}}{R_f} = 0$$

$$\frac{v_{02} - 2v_{01}}{2R} + SCv_{02} - \frac{v_{02}}{R_f} = 0 \quad R_f = \frac{2R}{H\Delta}$$

$$v_{02}\left(\frac{1}{2} + SC - \frac{H\Delta}{2R}\right) = \frac{v_{01}}{R}$$

$$v_{02}\left(SCR - \frac{\Delta}{2}\right) = v_{01}$$

$$\therefore \frac{v_{02}}{v_{01}} = \frac{1}{SCR - \Delta/2}$$

Now: $\frac{v_{01}}{v_x} = \frac{-1}{SCR}$

$$\therefore L(S) = \frac{-1/SCR}{SCR - \Delta/2}$$

Characteristic equation $L(s) = 1$

$$\therefore S^2C^2R^2 - \frac{SCR\Delta}{2} + 1 = 0$$

$\therefore$ Poles are

~~#55 problem on last page~~

$$Q = \frac{1}{(2 - R_2/R_1)}$$

for $Q = \infty$ ~ poles on jω axis

$\sim R_2/R_1 = 2$

for poles in R.H.P $R_2/R_1 > 2$

17.12

From Fig (17.5) assuming resistance of limiting network is very low

At positive peak

$$v_O = \left(\frac{1 + 20.3\text{ K}}{10\text{ K}}\right) v_O = 3.03 v_I \qquad (1)$$

$$v_O - \left[\frac{R_5}{R_5 + R_6} \cdot (v_O - (-15))\right] - 0.7 = v_I \quad (2)$$

Now for $10V_{P-P}$ out

$$\hat{v}_O = 5\text{ V}$$

$$\hat{v}_I = \frac{5}{3.03} = 1.65\text{ V}$$

using (2) $R_5 = 1\text{ k}\Omega$

$$5 - \left(\frac{1}{1 + R_6} \cdot (V_o + 15)\right) - 0.7 = 1.65$$

$$\frac{20}{1 + R_6} = 2.65$$

$$R_6 = \frac{20}{2.65} - 1$$

$$R_6 = 6.5\text{ k}\Omega = R3$$

If $R_3 = R_6 = \infty$ from (2)

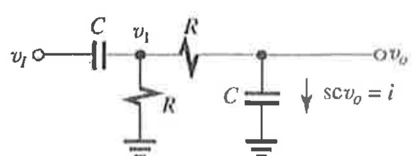
$$v_0 - \left(\frac{1}{1 + \infty}(V_0 + 15)\right) - 0.7 = \frac{v_0}{3.03}$$

$$v_0 - 0.7 = \frac{v_0}{3.3}$$

$$v_0 = 1.04\text{ V}$$

∴ oppoutput is $2\, v_0 = 2.08\ V_{p-p}$.

17.13



$$\frac{v_1 - v_0}{R} = SCv_0 \Rightarrow v_1 - v_0(1 + SCR)$$

ΣI at v_1

$$\frac{v_1}{R} + SC(v_1 - v_I) + SCv_0 = 0$$

$$v_0(1 + SCR) + SCR(v_0 + v_0 SCR) - SCRv_I + SCRv_0 = 0$$

$$v_0(1 + SCR + SCR + S^2C^2R^2 + SCR) = SCRv_I$$

$$\beta(s) \triangleq \frac{v_0}{v_I} = \frac{SCR}{S^2C^2R^2 + 3SCR + 1}$$

$$= \frac{1}{3 + SCR + 1/(SCR)}$$

From Fig(17.3) $A = 1 + R_2/R_1$

$$\beta(j\omega) = \frac{1}{3 + j\left(\omega CR - \frac{1}{\omega CR}\right)}$$

Zero phase when $\omega CR = \frac{1}{\omega CR}$

$$\omega = \frac{1}{CR}$$

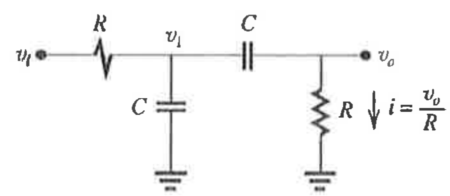
$$|\beta(W = 1/RC)| = \frac{1}{3}$$

for oscillations $1 + R_2/R_1 \geq 3 \Rightarrow \frac{R_2}{R_1} \geq 2$

$$L(s) = A\beta = \frac{1 + R_2/R_1}{3 + SCR + SCR}$$

$$L(jw) = \frac{1 + R_2/R_1}{3 + j\left(WCR - \frac{1}{WCR}\right)}$$

17.14



$$SC(v_1 - v_0) = \frac{v_0}{R}$$

$$v_1 = v_0\left(1 + \frac{1}{SRC}\right)$$

ΣI at v_1 :

$$\frac{-v_I + v_1}{R} + SCv_1 + \frac{v_0}{R} = 0$$

← SUB FOR v_1 and mult by $\frac{1}{SC}$

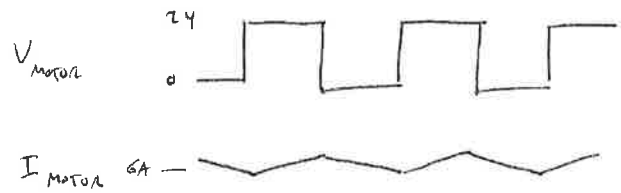
$$\frac{1}{SCR}[-v + v_0(1 + SCR)] + v_O\left(1 + \frac{1}{SCR}\right) + \frac{v_O}{SCR} = 0$$

$$v_O\left[\frac{1}{SCR} + 1 + \frac{1}{SCR} + \frac{1}{S^2C^2R^2} + \frac{1}{SCR}\right]$$

$$= \frac{v_I}{SCR}$$

Problem F, Solution

a) The DC voltage across the inductor and resistor is 12 V, so the DC current=12V/2Ω =6A.



b)

I_{DC}=6A (Assume this is constant over one period; this is equivalent to taking only the first term of the Taylor Series).

When V_{motor}=24V: $V_{Inductor}$=24-$V_{Resistor}$=24-6A*2Ω=12V $\quad L\frac{di}{dt} = V_{Inductor} = 12; \quad \frac{di}{dt} = \frac{V_{Inductor}}{L} = 6000A/s$

One half period = 0.5/12800=39µS=0.039mS. $\quad \text{ripple} = di = \frac{V_{Inductor}}{L}dt = 6000A/s \cdot 0.039mS = 0.23A$

V_{DC} = 2.4 V $\quad$ I_{DC} = 2.4/2 = 1.2 A

c)

When V_{motor}=24V $\quad V_{Inductor}$=24-$V_{Resistor}$=24-1.2A*2Ω=21.6V $\quad L\frac{di}{dt} = V_{Inductor} = 21.6; \quad \frac{di}{dt} = \frac{V_{Inductor}}{L} = 10800A/s$

Time high = 0.1/12800=7.8µS $\quad \text{ripple} = di = \frac{V_{Inductor}}{L}dt = 10800A/s \cdot 7.8\mu S = 0.084A$ (so… ripple is lower than with 50% duty cycle).

Check answer – ripple should be the same magnitude but opposite direction when V_{motor}=0 (current is dropping).

When V_{motor}=0V $\quad V_{Inductor}$=0-$V_{Resistor}$=0-1.2A*2Ω=-2.4V $\quad L\frac{di}{dt} = V_{Inductor} = -2.4; \quad \frac{di}{dt} = \frac{V_{Inductor}}{L} = -1200A/s$

Time low = 0.1/12800=70µS $\quad \text{ripple} = di = \frac{V_{Inductor}}{L}dt = -1200A/s \cdot 70\mu S = -0.084A$ (This verifies original answer).

d) I_{DC}=1A=(Vbatt/2-Eb)/2Ω=1A

$V_{Inductor}$=24-Eb-$V_{Resistor}$=24-10-1A*2Ω=12V $\quad L\frac{di}{dt} = V_{Inductor} = 12; \quad \frac{di}{dt} = \frac{V_{Inductor}}{L} = 6000A/s$

One half period = 0.5/12800=39µS=0.039mS. $\quad \text{ripple} = di = \frac{V_{Inductor}}{L}dt = 6000A/s \cdot 0.039mS = 0.23A$

So… the ripple is the same amount, but a much larger amount as a percent of the average current.

Problem G, Solution

a) Cutoff frequency: $\omega o=1/(R*C)=4.17E3$rad/sec, fo= 660Hz. Note: 660Hz<<12.8kHz (the pwm frequency)

b) Low pass filter, we only get DC component (660Hz<<12.8kHz). So v-=0.5 volts.

c) By the same argument: v-=0.1 volts.

d) v+=5.2*240/(20240)=0.0617V

e) v- = average of the voltage across J1. Therefore, v-=0.0617=10A*R_{J1} R_{J1} =0.00617Ω

f) The voltage at v+ is small, and therefore susceptible to noise. The capacitor forms a lowpass filter (fo=1/2*pi*C*(10k||(10k+240))≈ 300Hz) that will attenuate noise on the supply (5.2V) from affecting the voltage on v+.