It is easy to create a square wave of a precisely given frequency (e.g. using TIMER A on an MSP430). It is much harder to do so for a sine wave. The sine output oscillators we have considered oscillate at a frequency determined by several resistors and capacitors whose values are generally not known precisely, and are certainly not easy to change while maintaining precisely know values. This circuit (from "μP-Controlled Oscillator Delivers Rock-Bottom Distortion" at http://electronicdesign.com/analog/p-controlled-oscillator-delivers-rock-bottom-distortion) attempts to solve this problem and is shown below:

Figure it out.
Do all problems. Problems 1-9 are 10 points each, problem 10 is 5 points.

**Problem 1** The LTC1441 is a comparator with input at v- (pin 4) in this circuit, whose output is either 5 volts (high) or -5 volts (low). The circuit is set up with hysteresis. Find the two threshold voltages (i.e., find v+ (pin 3) when the output is high and when it is low).

\[ \pm 5 \cdot \frac{100}{10k} = \pm 49.5 \text{mV} \]

Small amount of hysteresis to keep square wave output clean.

**Problem 2** The input to the Schmitt trigger is the signal labeled BPA and is filtered by the 100\(\mu\)F capacitor and the 10k\(\Omega\) resistor. Note: the node labelled BPA in the upper left is connected to the one on the lower right.

i. What is the cutoff frequency (in Hz) of the filter formed?

\[ \omega_0 = \frac{1}{10k \cdot 100\mu F} = 1 \text{rad/s} \cdot 6 = .159 \text{Hz} = \frac{\text{passes all but very low}}{\text{pass}} \]

ii. Is it high-pass or low pass?

**Problem 3** If the output of the comparator is a square wave at ±5 volts, what is the voltage at the lower end of R1A? Assume a constant 0.6 volt drop for the diodes when they are on.

\[ \pm 0.6 \text{V} \]
Problem 4) The transfer functions below are for highpass, bandpass, lopass and notch. Identify which is which and explain your answer (no more than one brief sentence per filter).

\[
H_1(s) = H_{0,1} \frac{\omega_0^2}{s^2 + \omega_0^2 s + \omega_0^2} \\
\downarrow \\
\omega \rightarrow 0 \quad 1 \\
\omega \rightarrow \infty \quad 0
\]

\[
H_2(s) = H_{0,2} \frac{s^2 + \omega_0^2}{s^2 + \omega_0^2 s + \omega_0^2} \\
\downarrow \\
\text{notch} \\
1 \\
0
\]

\[
H_3(s) = H_{0,3} \frac{s^2}{s^2 + \omega_0^2 s + \omega_0^2} \\
\downarrow \\
\text{lowpass} \\
1 \\
1
\]

\[
H_4(s) = H_{0,4} \frac{\omega_0 s}{s^2 + \omega_0^2 s + \omega_0^2} \\
\downarrow \\
\text{band} \\
0 \\
0
\]

The circuit shown on the first page has a square wave input at CLK (upper right) that drives the switched-capacitor filters and determines \(\omega_0\).

Problem 5) Use the datasheet and show that the left side of the circuit implements a bandpass (if BPA is output) or lowpass filter (LPA is output). \(V_{in}\) is shown at right (bottom-left). There is no need to do any derivations. What are the values of Q, \(\omega_0\), \(H_{0,LP}\), and \(H_{0,BP}\)? (Assume you know the clock frequency)

\[
\omega_0 = \frac{f_{CLK}}{100} \\
Q = \frac{R_{2A}}{R_2} = 10 \\
H_{0,LP} = -\frac{R_2}{R_1} \omega_0 \frac{1}{2} \\
H_{0,BP} = -\frac{R_3}{R_1} = -1
\]
Problem 6) The parts of the circuit examined so far implement an active filter tuned oscillator like the one shown in the block diagram (the bandpass output, BPA, drives a comparator; the comparator, in turns, drives the bandpass input). However in the block diagram, the comparator does not invert (there is a high output for high input and vice versa). Our comparator inverts the input; how can the circuit still work properly?

\[
\text{since } H_{\text{HP}} \text{ invert (} H_{\text{HP}} \text{ is negative) the comparator must invert.}
\]

One of the main factors in the performance of such a filter is the suppression of harmonics in the square wave output of the comparator. The rest of the circuit (and exam) is devoted to this task.

Problem 7) Derive (don’t just look up) the relationship between LPA (the input) and HPB (the output), and LPA and LPB (a second output). Note: the LPA here is connected to the one on the left side of the original diagram. The output of the left side (labeled "A") is the input for the right side (labeled "B"). The next page is blank for your work.

\[
Q = 1.005 \left(\frac{R_3}{R_2}\right) F_{\text{clk}} \left(1 + \frac{R_3}{R_2}\right)
\]
\[ H_p = \frac{1}{r_1} - \frac{V_{in}}{r_1} \frac{R_2}{r_1} + \frac{V_{in}}{r_3} \frac{R_3}{r_3} + \frac{L_p}{r_4} \]

\[ L_p = H_p \frac{w_1}{5} \]

\[ L_p (1 + \frac{w_1}{5} \frac{R_2}{r_2} + \frac{(w_1)^2}{5} \frac{R_3}{r_3}) = -\frac{V_{in}}{5} \frac{w_1}{R_1} \]

\[ \frac{H_p}{V_{in}} = \frac{R_2}{r_1} - \frac{1}{1 + \frac{w_1}{5} \frac{R_2}{r_3} + \frac{(w_1)^2}{5} \frac{R_3}{r_4}} \]

\[ w_0 = \sqrt{\frac{R_1}{r_1}} \]

\[ \frac{w_0}{Q} = \frac{w_1}{\sqrt{R_1 r_4}} \frac{1}{Q} = \frac{w_1}{R_3} \]

\[ Q = \sqrt{\frac{R_2}{r_2}} \frac{R_2}{r_3} \approx 2.05 \]

\[ H_{0.1p} = -\frac{R_2}{r_1} \approx -0.98 \]

\[ \frac{L_p}{V_{in}} = -\frac{R_2}{r_1} \frac{w_1}{5} \frac{1}{1 + \frac{w_1}{5} \frac{R_2}{r_3} + \frac{(w_1)^2}{5} \frac{R_3}{r_4}} \]

\[ H_{0.1p} = \frac{w_1}{R_1} \]

\[ B_0 = H_{0.1p} \frac{w_1}{R_1} \frac{R_2}{r_4} \]

\[ H_{0.1p} = -\frac{R_4}{r_1} \approx -0.98 \]
Problem 8) Derive (don't just look up) the relationship between the output (labeled "sine output") and LPA. How does this help make the output of the circuit more like a pure sine wave?

Note: you can ignore the 1200 pF capacitor and the 4.99 k resistor (the resistor is there to minimize effects of bias current - we talked about that in the last class).

\[ v_o(t) = \left( L \frac{R_L}{R} + \frac{d}{dt} \frac{R_L}{R_H} \right) \left( \frac{1}{\pi^2 + 1} \right) \left( \frac{1}{\frac{s^2 + \frac{1}{Q^2}}{\omega_0^2}} \right) \left( \frac{\omega_0^2}{s^2 + \frac{1}{Q^2} + \omega_0^2} \right) \]
Problem 9) Why is the output of LPA chosen to feed the final filtering stages rather than the output of BPA?

as \( \omega \to 0 \) drop off is \(-40 \text{ dB/dec} \) w/ Lopass

but only \(-20 \text{ dB/dec} \) w/ Bnpass.

Lopass will attenuate harmonics better.

Problem 10) (5 points) Speculate on the purpose of the 1200 pF capacitor.

output of switched cap circuit is stair step

1200 pF sets a cutoff (13.3 kHz) to smooth out steps.