Lecture 4: SolidWorks 3 – Assemblies / Matrices

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Course web page: http://www.swarthmore.edu/NatSci/echeeve1/Class/e5/E5Index.html
Remember...

- Wizards available Tuesdays 8:30-10:00 and Wednesdays 8:00-10:00.
- Thursday 9/24: Bridge Design
- Thursday 10/1: Matrix Assignment
- Thursday 10/1: SolidWorks Mechanism (this week’s lab)
- Thursday 10/8: Bridge report (on wiki) is due.
The big picture...

- Getting ready for midterm project – a mechanical laser pointing system.
- Today... SolidWorks Assemblies
- Today... Matrices
- Next several week – MatLab

- Also today – Nancy Burkette, career services.
...SolidWorks demo...
Vectors and Matrices

- Vectors and matrices are an efficient notational method for representing lists of numbers.
Vectors

- A typical vector might represent the high temperature every day for a week (a row vector)

\[
\text{HighTemp} = \begin{bmatrix} 25 & 32 & 33 & 38 & 43 & 45 & 41 \end{bmatrix}
\]

- ...or as a single column (a column vector):

\[
\text{HighTemp} = \begin{bmatrix} 25 \\ 32 \\ 33 \\ 38 \\ 43 \\ 45 \\ 41 \end{bmatrix}
\]

- In both cases the vector has seven elements that can be individually referenced by their index.

\[
\text{HighTemp}(4) = 38
\]

- A vector is always a single row or column.
2D Matrices (1)

- A matrix (singular of matrices) is (for our purposes) a series of numbers listed in two or more dimensions. We will limit ourselves to two dimensions this week.
- High temperatures collected over a 28 day period (4 weeks).

\[
\begin{bmatrix}
25 & 32 & 33 & 38 & 43 & 45 & 41 \\
42 & 43 & 45 & 46 & 48 & 41 & 39 \\
39 & 41 & 43 & 47 & 48 & 48 & 47 \\
50 & 49 & 45 & 48 & 50 & 51 & 53
\end{bmatrix}
\]

- Two indices to specify an element of the matrix (row first, then column).
- For example the high temperature on the 2\(^{nd}\) week, 3\(^{rd}\) day is HighTemp(2,3) and is equal to 45. Note that the index for the row comes first.
2D Matrices (2)

- We could also write the matrix with rows and columns interchanged. This is referred to as taking the "transpose" of the matrix.

\[
\text{HighTemp} = \begin{bmatrix}
25 & 42 & 39 & 50 \\
32 & 43 & 41 & 49 \\
33 & 45 & 43 & 45 \\
38 & 46 & 47 & 48 \\
43 & 48 & 48 & 50 \\
45 & 41 & 48 & 51 \\
41 & 39 & 47 & 53 \\
\end{bmatrix}
\]
2D Matrices (3)

- In general, a matrix consisting of \( m \cdot n \) elements can be arranged in \( m \) rows and \( n \) columns, yielding an \( m \times n \) (read \( m \) by \( n \)) matrix, which we'll call \( A \).

\[
A = \begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{1n} \\
    a_{21} & a_{22} & \cdots & a_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}
\]

- The symbol \( a_{ij} \) represents the number in the \( i^{\text{th}} \) row and the \( j^{\text{th}} \) column. Row comes first, followed by column.
2D Matrix operations

Consider \[ A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix} \]

- **Equality**: All elements are equal
  - \( A = B \)
  - \( A \neq C \)
- **Addition**: Matrices must be same size. Add corresponding elements.
  \[ A + C = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ 3 & 5 \end{bmatrix} \]
- **Operations with a scalar**: Matrix can be multiplied by a scalar (a single number) by multiplying each element of the array by that number.
  - For example: \( \text{CelcTemp} = (\text{HighTemp} - 32) \times \frac{5}{9} \)
  - The same can be done with addition, subtraction or division by a scalar.
Multiplications of vectors

- A row vector can be multiplied by a column vector, in that order, to yield a scalar if and only if the have the same number of elements.

- In general:

\[
\mathbf{u} = u_1 \quad u_2 \quad \cdots \quad u_n , \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}
\]

\[
\mathbf{uv} = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n , \quad \text{or} \quad \mathbf{uv} = \sum_{i=1}^{n} u_i v_i
\]

- Example:

\[
1 \quad 3 \quad 5
\begin{bmatrix}
2 \\
4 \\
6
\end{bmatrix}
= 1 \cdot 2 + 3 \cdot 4 + 5 \cdot 6 = 44
\]
Multiplication of two matrices

To multiply $A$ and $C$...

- The number of columns in $A$ equals the number of rows in $C$ (the inner dimensions are equal).
- If $D = AC$, $A$ is $m \times n$, $C$ is $n \times p$, $D$ is $m \times p$.
- $d_{ij}$ is row $i$ of $A$ by column multiplied by column $j$ of $C$.

$$d_{ij} = \sum_{k=1}^{n} a_{ik} c_{kj}$$

- This is easier with an example...
Matrix multiplication (1)

\[
\begin{bmatrix}
1 & 4 \\
2 & 3 \\
\end{bmatrix}
\begin{bmatrix}
0 & 3 \\
1 & 2 \\
\end{bmatrix}
= \begin{bmatrix}
1 \cdot 0 + 4 \cdot 1 & 1 \cdot 3 + 4 \cdot 2 \\
2 \cdot 0 + 3 \cdot 1 & 2 \cdot 3 + 3 \cdot 2 \\
\end{bmatrix}
= \begin{bmatrix}
4 & 11 \\
3 & 12 \\
\end{bmatrix}
\]

\[d_{11} = 1 \cdot \begin{bmatrix}
0 \\
1 \\
\end{bmatrix} = \text{1st row of } A \times \text{1st column of } C.
\]

\[= 1 \cdot 0 + 4 \cdot 1 = 0 + 4 = 4
\]

\[= \text{1st row, 1st column of } D
\]

\[d_{12} = 1 \cdot \begin{bmatrix}
3 \\
2 \\
\end{bmatrix} = \text{1st row of } A \times \text{2nd column of } C.
\]

\[= 1 \cdot 3 + 4 \cdot 2 = 3 + 6 = 11
\]

\[= \text{1st row, 2nd column of } D
\]

\[d_{21} = 2 \cdot \begin{bmatrix}
0 \\
1 \\
\end{bmatrix} = \text{2nd row of } A \times \text{1st column of } C.
\]

\[= 2 \cdot 0 + 3 \cdot 1 = 0 + 3 = 3
\]

\[= \text{2nd row, 1st column of } D
\]

\[d_{22} = 2 \cdot \begin{bmatrix}
3 \\
2 \\
\end{bmatrix} = \text{2nd row of } A \times \text{2nd column of } C.
\]

\[= 2 \cdot 3 + 3 \cdot 2 = 6 + 6 = 12
\]

\[= \text{2nd row, 2nd column of } D
\]
Matrix multiplication (2)

- Multiplication is not generally commutative

\[
\begin{align*}
AC &= \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 11 \\ 3 & 12 \end{bmatrix} \\
CA &= \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 5 & 10 \end{bmatrix} \\
\end{align*}
\]

\[
\begin{bmatrix} 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = 44
\]

- There is no matrix division.
Special Matrices

- **Identity Matrix:** a square matrix has one's along the main diagonal, and 0's elsewhere. $AI = IA = A$

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

- **Inverse Matrix:** If the matrix $A$ has an inverse $G$, we write $G = A^{-1}$, and $GA = AG = I$. Note that some matrices don't have inverses.
Examples (1)

\[ A = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad U = 1 \quad 2, \quad V = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \]

What is \( a_{12} \)?

What is \( a_{21} \)?

What is \( A+B \)?

What is \( AB \)?

What is \( UV \)?

(Note, in this case \( AB = BA \), though multiplication isn't generally commutative. Also... B and A are inverses of each other).

What is \( AI \)?

What is \( BA \)?

(Note, in this case \( IA = AI \), though multiplication isn't generally commutative).

What is \( IA \)?

What is \( VU \)?

(Note, in this case \( UV \neq VU \), because multiplication isn't generally commutative).
How could you use matrix multiplication to find the average of each row of the matrix $A$?

How could you use matrix multiplication to find the average of each column of the matrix $A$?
Examples (1)

What is $a_{12}$?  -1

What is $a_{21}$?  1

What is $A+B$?
\[
\begin{bmatrix}
\frac{2}{3} & -\frac{2}{3} \\
\frac{2}{3} & \frac{1}{3}
\end{bmatrix}
\]

What is $UV$?  $1 \times 3 + 2 \times 4 = 11$

What is $AI$?
\[
\begin{bmatrix}
1 & -1 \\
1 & 2
\end{bmatrix}
\]

What is $IA$?
\[
\begin{bmatrix}
1 & -1 \\
1 & 2
\end{bmatrix}
\]

What is $BA$?
\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

What is $AB$?
\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

(Note, in this case $AB=BA$, though multiplication isn't generally commutative. Also... B and A are inverses of each other).

What is $VU$?
\[
\begin{bmatrix}
3 & 6 \\
4 & 8
\end{bmatrix}
\]

(Note, in this case $UV \neq VU$, because multiplication isn't generally commutative).

$A = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$,  $B = \begin{bmatrix} 2 & 1 \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$,  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,  $U = 1 \ 2$,  $V = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
Examples (2) 

\[ A = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad U = 1 \quad 2 \quad \text{,} \quad V = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \]

How could you use matrix multiplication to find the average of each row of the matrix \( A \)?

\[
\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 3/2 \end{bmatrix}
\]

How could you use matrix multiplication to find the average of each column of the matrix \( A \)?

\[
\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \end{bmatrix}
\]