E77 Exam 1, 2005

Name: ________________________________ (10 pts)

Answer all of the problems. All count equally towards your grade. Ask me if anything is unclear.

Problem 1.
Consider a square prism of material that is of width "w" on each side and of depth "d", as shown.

Recall that $E = \frac{dV}{dx}$, $J = \sigma E$, $I = J \cdot A$, where V is voltage, E is electric field, J is current density, $\sigma$ is conductivity, A is area and I is current.

a) Show that the resistance between the front edge (the skinny "w" by "d" rectangle with thick black lines – nearest to the bottom of the page) and the back edge (the skinny "w" by "d" rectangle nearest to the top of the page) is constant regardless of the dimension "w." If this is unclear, please ask me.

$$I = \sigma E A = -\sigma \frac{dV}{dx} \cdot w \cdot d$$

$$= \sigma \frac{V_1 - V_2}{w} \cdot w \cdot d$$

$$= \sigma d (V_1 - V_2)$$

if front is at $V_1$ (x=0)

back is at $V_2$ (x=0)

$$\frac{dV}{dx} = \frac{V_1 - V_2}{d - w} = \frac{V_1 - V_2}{w}$$
Because the resistance is independent of the size of the square of material, resistance of various layers is often given in ohms/square, Ω/□, or R_0; this is also called sheet resistance. For example in our 2μ semiconductor process, for polysilicon R_0=21 Ω/□ and for metal1 R_0=0.06Ω/□.

b) If the doping of n-type and p-type semiconductor are the same, would you expect the resistance of a square of n-type silicon or p-type Silicon to be higher? Why?

\[ p\text{-type} \rightarrow \text{mobility is lower.} \]

e) (unrelated to previous) Describe, in one or two sentences, how current flows in a p-type semiconductor keeping in mind that no positive charges actually contribute to the current.

Acceptor dopant takes a neighboring electron to fill covalent bond ⇒ a hole results
In a semiconductor we often used the following expression for current density:

\[ J = q \cdot p \cdot \mu_p \cdot E + q \cdot n \cdot \mu_n \cdot E + q \cdot D_n \cdot \frac{dn}{dx} - q \cdot D_p \cdot \frac{dp}{dx} \]

d) (unrelated to previous) Briefly explain the physical basis of each term, and also describe why only the last term has a minus sign.

For drift terms (first two), electrons + holes move in different directions, so current is in same direction under influence of E-field.

For diffusion, electrons + holes move in same direction, so current is opposite due to concentration gradient.

To make a resistor, it is sometimes required to use many square of material. Instead of a single line, the material is often made to snake back and forth as shown. This image depicts a resistance between the points x and y made by many squares formed into a zig-zag pattern.

e) (unrelated to previous) Consider three squares laid out as shown in the two images below; one is in a straight line, one forms a 90° bend (of which there are many in the example above). Which of the two structures would you expect to have lower resistance between points A and B? Why?

Typically the corner square is taken to be from 0.5 to 0.65 of the resistance of the non-corner squares.
Problem 2: Consider the circuit shown, where the transistor is a standard Silicon transistor with $I_s = 1 \text{pA}$, $n = 1$. There are two equations which define the circuit ($v_d = \text{voltage across diode, } i_d = \text{current through diode}$).

\[ i_d = I_s \left( e^{v_d/0.025} - 1 \right), \text{ and} \]
\[ i_d = \frac{(V_1 - v_d)}{R_a} \]

a) These two functions are plotted below. What are $V_1$ and $R_a$?

\[ V_1 = 3 \]
\[ R_a = 1 \text{ k} \Omega \]

b) Where the two graphs intersect is the operating point of the circuit. Describe how you would use the Newton-Raphsen technique to find the intersection of the two graphs.

\[ f = \frac{V_1 - v_d}{R_a} - I_s \left(e^{v_d/0.025} - 1\right) \]
\[ V_1 = V_0 = \frac{f(V_0)}{f'(V_0)} \]
If the standard diode is replaced by a tunnel diode, a very different situation occurs. The current voltage characteristic for a tunnel diode is shown below along with the linear current-voltage relationship for the resistor and voltage source.

In this graph, there are three possible solutions. Only the leftmost and rightmost are physically reasonable; the middle solution is physically unreasonable and doesn’t occur in practice. The reason for this is described below – feel free to skip this, you don’t need to understand to complete the problem.

c) Describe how the Newton-Raphsen method could be applied to this problem and clearly state (depending on the initial guess) whether or not the Newton-Raphsen method might converge to the middle solution.

Any solution is possible given N-V, 

\[ f(u_d) \neq f(u_r) \Rightarrow \]

(You may skip this description during exam) Assume the current is operating at the leftmost crossing of the two curves. If the diode voltage decreases a little bit, the current through it will tend to decrease as well; this will act to increase the current through the resistor, so the operating point stabilizes where the graphs cross. If the diode is operating at the middle point where the two curves cross, then if the diode voltage decreases a little bit, the current through it will actually increases (this is, in effect, a negative resistance), this increases the current through the resistor (and the voltage across it), which further decreases the voltage across the diode... – this process continues until the circuit ends up at a stable operating point.
d) For the possible solutions from the Newton-Raphsen method roughly (very roughly) estimate the range of initial guesses that will end up at the various possible solution (e.g., what range of initial guesses will converge to the leftmost solution). Your estimate can be rough, but be clear in your reasoning.

e) (unrelated to previous) The Newton-Raphsen technique can also be used for other purposes. One well-known technique is to find the square root of numbers. Use the Newton-Raphsen technique to find the square root of 2, starting with an initial guess of 1. Explain your technique and give the result of the first five iterations.

\[
f(x) = x^2 - 2 \quad \frac{df}{dx} = 2x
\]

\[x_{i+1} = x_i - \frac{x_i^2 - 2}{2x_i} = \frac{1}{x_i} + \frac{x_i}{2}\]

\[
\begin{align*}
x_0 &= 1 \quad &x_1 &= 1.5 \\
x_0 &= 1.5 \quad &x_1 &= 1.42 \\
x_0 &= 1.42 \quad &x_1 &= 1.414216 \\
x_1 &= 1.414214 \\
\sqrt{2} &\approx 1.414214
\end{align*}
\]
**Problem 3:** NMOS Invertor with Passive Load

Consider the circuit shown which is one circuit of many that are integrated onto a single substrate. The resistor is formed of various squares of some material as described in Problem 1. The transistor is NMOS with $V_t = 1V$, $k' = 50\mu A/V^2$, $W = 6\mu$ and $L = 2\mu$. $V = 5V$, $R_1 = 10\Omega$

\[ A \leq V_{in} \uparrow, I_d \uparrow, V_o \downarrow \]

**a)** Argue that this will work as an inverter for an appropriate value of $R_1$.

**b)** Determine the range of input voltage for which:

- i) the transistor is off?
- ii) the transistor is ohmic?
- iii) the transistor is saturated?

\[ \text{i) off for } V_{in} < V_T \implies \text{ then goes to saturation b/c } \]

\[ V_{DS} \text{ is large.} \]

\[ \text{ii) saturated } \]

\[ I_D = \frac{k'W}{2L}(V_{GS}-V_T)^2 = 100 \times 10^{-6}(V_{in}-1)^2 \]

\[ V_o = 5 - R_1(100 \times 10^{-6}(V_{in}-1)^2) = 5 - (V_{in}-1)^2 \]

\[ \text{At edge of saturation } \]

\[ V_o = V_{DS} = V_{DS} = V_T - V_T = V_{in} - V_T \]

\[ (V_{in}-1) = 5 - (V_{in}-1)^3 \implies (V_{in}-1)^2 + (V_{in}-1) - 5 = 0 \]

\[ (V_{in}-1) = \frac{-1 \pm \sqrt{1 + 20}}{2} \]

\[ \text{Takes positive } \]

\[ \text{Saturated } V_{in} > 1, V_{in} < 1.79 \]

\[ \text{ohmic } \frac{V_{in}}{2} \leq V_{in} \leq 2 + 0 \]

\[ V_{in} = 2.0 + 0.9 \implies 2.9 \]
c) Give an expression (or several) for $V_{out}$ as a function of $V_{in}$.

\[
\text{off } \Rightarrow \quad V_c < 1 \quad \Rightarrow \quad V_o = V_{dd} = 5V \\
\text{Saturation } \Rightarrow 1 < V_c < 1.75 \quad \Rightarrow \quad V_o = V_{dd} - (V_c - 1)^2 \\
\text{Ohmic } \Rightarrow 1.75 < V_c \quad \Rightarrow \quad \begin{aligned}
V_o &= V_{dd} - k'\frac{W}{L} \left((V_c - V_t) V_{bs} - \frac{V_{bs}^2}{2}\right) \\
V_o &= 5 - 2(1(V_c - 1))V_o - \frac{V_o^2}{2}
\end{aligned} \\
V_o &= 5 - 2(V_c - 1)V_o + V_o^2 \\
\Theta &= V_o^2 + (1 - 2V_c)V_o + 5 \\
\text{Take } &- \\
V_o &= \frac{2V_c - 1 \pm \sqrt{(2V_c - 1)^2 - 4}}{2}
\]

Check: \[
SAT \quad V_c = 2.79 \quad V_o = 3.8 \\
\text{Ohmic } V_c = 2.79 \quad V_o = 1.8
\]

d) Determine a value of $R_1$ so that $V_{OL} = 0.5$ V. If polysilicon were used as the material for the resistor, approximately how many squares would be required ($R_d = 21 \, \Omega$)? (This is why in NMOS logic the resistor is replaced by another transistor).

\[
I = 5 - \frac{k'W}{L} R \left((5 - 1) 1 - \frac{1}{2} \right) = 5 - 2 \times 10^4 \times 5 (3.5) \\
R = 5.71 \, k \Omega = 272 \, \text{squares}
\]

e) Why is the substrate grounded? What could happen if it was not grounded?

If not grounded, current could enter source from other devices on Si (through the substrate).