Interval exchange transformations from tiling billiards

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Joint work with:
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Elijah Fromm, Sumun Iyer and Paul Baird-Smith
in the SMALL REU at Williams College
Conjecture:
Enclosed vertices & edges form a tree

Theorem:
Any trajectory enclosing a tree has period $4n+2$
Start with triangle and trajectory
Add circumscribing circle
Flip: angles CCW
Rotate: head to 0
Flip: angles CCW
Rotate: head to 0
Next triangle!
Fold so trajectory matches
Flip: angles CCW
Rotate: head to 0

\[ X' = \tau - 2\gamma - X \]
Our Interval Exchange Transformation (IET) is defined by:

\[ X' = \begin{cases} 
\tau + 2\beta - X & \text{if } 0 < X < 2\beta \\
\tau + 2\beta - 2\gamma - X & \text{if } 2\beta < X < 2\beta + 2\gamma \\
\tau - 2\gamma - X & \text{if } 2\beta + 2\gamma < X < 2\pi 
\end{cases} \]

\[ X' = \tau - 2\gamma - X \]

… an orientation-reversing IET.
Tiling billiards IET

Interval **lengths**: angles

**Shifts**: angles & trajectory

Starting **point**: orientation of triangle
A short advertisement for flipped IETs
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Everything is flipped periodic, every point is stable, periods of form 4n+2
Comparison to non-flipped IETs

If $|AB|$ and $|C|$ are irrationally related, every point is aperiodic.
Comparison to non-flipped IETs

If $|AB|$ and $|C|$ are irrationally related, every point is aperiodic.
- Switch a&b, c&d, e&f
- Rotate a half turn
Rauzy fractal
from Hooper & Weiss
Trihexagonal tiling collaborator

Pat Hooper (CCNY)
Speaking here on January 15
Glue rhombuses to get an infinite translation surface that is a cover of the torus.
Two types of periodic behavior:

- Periodic directions
- Drift-periodic directions
Theorem: **Periodic** and **drift-periodic** directions are those in the triangle grid.