My core questions

How do students learn? How should we teach? These are the driving questions of my life as an educator. I am always thinking about how to best use the precious three sessions of 50 minutes per week that I have with my students.

Three ways to teach

Sequential lecture. When I lecture a calculus course, it goes like this:

Day 5: Limits. Motivation, definition, example, example, application.
Day 6: Derivatives. Motivation, definition, example, example, application.
Day 7: Differentiability. Motivation, definition, example, example, application.

Sequential problems. Instead of telling the students about those topics, I could give them problems where they learn the material by solving the problems.

Day 5: Problems about limits.
Day 6: Problems about derivatives.
Day 7: Problems about differentiability.

Integrated problem sets. Instead of doing all of those problems for one topic on one day and then moving on, layer the topics to build understanding over time.

Day 5: Limits examples, definition of derivatives, differentiability motivation.
Day 6: Derivative examples, differentiability definition, partial derivative motivation.
Day 7: Differentiability examples, definition of partial derivatives, gradient motivation.

I have found that using integrated problem sets like this works well for student engagement, motivation, and retention of understanding, so this is what I do.

Is it the Moore method?

No. I write the textbook, which consists of problems that build the curriculum a little at a time. The students solve the problems and discuss their solutions.
My best courses are problem-centered and discussion-based

I am passionate about inquiry-based, student-centered learning. I learned math in this way — my high school curriculum consisted entirely of word problems, carefully constructed to build ideas. I have taught high school students in this way, I have taught high school teachers to teach in this way, and I have written my own inquiry-based curricula and taught classes this way at every opportunity, in eight different courses at Northwestern University, Williams College and Swarthmore College.

I loved learning with this method as a student, and I love teaching with this method as an instructor. Preparing for class is fun and intellectually challenging for me, as I figure out how to write homework problems that build ideas a little at a time, and construct homework assignments (see example at right) that each incorporate many skills and ideas.

Class time is fun for everyone, as the students work together, ask each other questions, and grow in their ability to articulate their ideas to their peers.

How I like to run classes

In my classes, there is daily homework, such as the page above. As soon as students enter the classroom, they choose a problem from the homework and write up their solution on the board, often in collaboration with other students (see below).
Class time consists of each pair or group of students in turn presenting their (complete or partial) solution to each problem, to the rest of the class. The other students ask questions about the solution, suggest alternative methods, correct errors, and so on. Sometimes a question is a simple extension of a previous idea and the discussion takes only a few minutes, while other times students ask challenging questions and the discussion of a single problem can take 15 minutes or more.

I tailor the class format to each course. When I taught a senior seminar at Williams, the 25 students in the course were randomly assigned each day to one of two different discussion groups. Here at Swarthmore, I have taught four different 16-student classes, and it has worked best to have 8-student discussion groups (see photo to the right). In my 26-student class, students work in six groups of 4-5 students each. In all cases, students often report that they look forward to coming to class, they never look at the clock, and they have a good sense of community with their classmates.

Problem-based curriculum builds curiosity and growth mindset

My goal is to teach students to enjoy working on challenging problems – and it works! At Northwestern University, I taught one of three sections of honors multivariable calculus in a discussion format, where the other two sections were taught in a lecture format with a problem-solving session once a week (which I also instituted, as the course head). At the beginning of the spring quarter at Northwestern, 60% of the students in my discussion course said they would prefer to work on a problem that they didn’t know how to solve, and by the end of the 9-week quarter, it had increased to 80% (see chart above). By contrast, the percentage in the lecture classes did not change much. I have since replicated this finding in my other discussion-based and lecture-based courses. For discussion of statistical significance and much more data, see my paper Inquiry-based learning in a first-year honors course [D18].
Discussion-based classes build skills and engage students

The students in discussion-based classes are more engaged than those in lecture-based classes. You can see this in their posture, as they are paying attention to their peers’ explanations and questions. I have measured this in how often they self-report texting in class (less often in discussions), and in how often they report missing class (see chart above). I have also heard this in their casual conversations, as they mention that our class is their most fun class of the semester!

Students learn just as much mathematics

Some people worry that less material is covered under the discussion method than under the lecture method. At Northwestern, I made sure that my multivariable calculus curriculum matched the pace of the lecture sections, so that all of the students could take the midterms and the final exam together. The students’ exam performance was essentially the same under each method. For details, see my paper [D18].

Students learn beyond the course material

I have been the primary instructor at Swarthmore, Williams, Northwestern, Brown and Phillips Exeter. At each of these institutions, the students are bright and motivated — they would learn with any teacher, and any method. My goal is to push them beyond just learning, to asking their own questions and shaping their own discovery.

I believe that a college course should teach more than the syllabus. Math courses usually teach, beyond the particular definitions and theorems, the skills of hard work, perseverance, and time management. Students in my discussion-based classes additionally report that they have learned “to be more articulate in discussing math and to be more confident in presenting my work to others,” and “how to not be satisfied with a right answer and always be looking for efficiency,” and my personal favorite, “how to ask good questions.” These students are getting an education.
Discussion celebrates diversity

When a classroom has students with a wide variety of backgrounds, it is difficult to figure out where to “pitch” the class. In a lecture class, all students receive the same content at the same pace. In a problem-centered, discussion-based method, the course meets each student where they are: students have as much time as they like to think about the homework problems the night before, and then each student can guide the direction of the class discussion to the places where they are having trouble, and to the ideas they want to discuss. I enjoy having students with different backgrounds, who have learned mathematics using different methods and terminology, as this forces students to really explain their solutions clearly and listen to each other, which are essential skills.

Teaching awards

Everywhere I have taught that had teaching awards, I have won them:

- Northwestern University Dept of Mathematics, award for Excellence in Teaching
- Brown University, nominated for Presidential Award for Excellence in Teaching
- Brown University Department of Mathematics, award for Excellence in Teaching
- Williams College, Morgan prize for teaching (as an undergraduate TA)

I can also lecture effectively

By now I have surely convinced you that I am passionate about teaching with a problem-based curriculum in student-centered classes. I am also realistic with respect to the constraints of a math department, and of my time. Before Northwestern University approved my proposal to teach a pilot discussion-based section of the honors multivariable calculus course (Math 290), I lectured in the course for seven quarters. My student evaluation scores are below.

I am proud that my scores for challenging the students intellectually (green line above) were, in every one of the seven quarters, the highest of all the four to six instructors of Math 290, who were themselves all teaching-track faculty. By the final winter of my postdoc, my scores on all six measures were among the highest in the
My effectiveness as a lecturer improved to one of the highest in the department (scale of 1 to 6) in a department that has a separate tenure line for teaching-track faculty that attracts phenomenal teachers.

**My classes in action**

I am constantly encouraging my colleagues to sit in on my classes, to see how the method works and to see my students in action. Since you will probably not be able to attend one of my classes, I have posted videos of one of my multivariable calculus classes from Northwestern. The students discuss problems about (links below):

- Volume of a three-dimensional region
- Area in polar coordinates
- Changing order of integration in a triple integral
- Triple integral estimation

If you sit in on my class or if you watch one of these videos, you will not see me doing anything particularly amazing. I help the students remember discussion skills such as speaking loudly or writing their ideas on the board, and I help with logistics of the document camera and screen. You will find that it is the *students* who are doing amazing things! Usually, my colleagues’ first question at the end of observing my class is, “How did you get the students to do that?” My job is to write engaging problems, and to create a classroom atmosphere of kindness, curiosity and learning.
The problem sets

All of my problem-based curriculum materials are freely available, and everyone is welcome to use them. You may wish to read through them, to see how the problems on each page build on previous problems, and how each night’s homework has a mix of several topics. The problem books are linked below.

- Billiards, Surfaces and Geometry (Williams)
- Real Analysis (Swarthmore)
- Introduction to Proofs (Northwestern)
- Multivariable Calculus (Williams)
- Multivariable Calculus (Northwestern)
- Discrete Mathematics (Swarthmore)
- Calculus 1 (Swarthmore)

Undergraduate research

I have mentored undergraduate research groups at two different REUs in three summers: Summer@ICERM at Brown University in 2012 and 2013, and SMALL at Williams College in 2016. Additionally, I had three senior thesis students, all Williams College class of 2017. Of all the students I have mentored, half are female and half are male, and two are members of underrepresented minority groups.

I enjoy working closely with students, helping them to find questions that excite them, encouraging them to explore and discover, and challenging them to write down their ideas rigorously.

My three REU groups all worked on the same “Tiling billiards” problem (see my research statement for details). The 2012 group explored the basic ideas of the system, the 2013 group expanded the 2012 group’s results and proved them in much greater generality while also coming up with observations and conjectures, and the 2016 group figured out an organizing principle of the entire system that proved the 2013 group’s conjectures and connected the problem to interval exchange transformations (IETs), a large area of dynamical systems research.

Year over year, the students’ work improved almost exponentially, which was exciting for them and for me. The paper from the 2013 group [DDRS18] has been published,
and has already been cited several times and extended, and the paper from the 2016 group [BDFI18] is on the arXiv and is submitted for publication.

My three thesis students at Williams worked on three very different projects. Megumi Asada, who is now a master’s student at Cambridge, studied periodic billiard paths on the hexagon, which extended classical results on the square billiard table and my results on the pentagon (see my research statement for details). Paul Baird-Smith, who is now in computer science graduate school at Texas, explored the possible configurations of systems of mechanical cranks, writing computer programs to model the behavior. Dylanger Pittman, who is now in math graduate school at Emory, worked on determining the least-perimeter way to enclose and separate two regions (the double bubble problem) in Borell space, for dimensions 1 and 2.

When my thesis students come to my office to discuss their progress, they are usually very excited to talk about their discoveries and explain their proofs. I love to help them on this journey, directing them to related papers, suggesting other ways of thinking about the problem, and generally making sure that they have enough space to take the project in the directions that interest them.

I have many undergraduate projects that I would like to have students work on in the future; see my research statement for details.

**My dream**

By the end of my career, I would like every math course, in every college and university, to use an inquiry-based method. Working toward making this happen gets me up in the morning, and inspires me every day.

I don’t want to see any more students, so bright and capable, sitting quietly in lectures. They may sit quietly while doing homework, and then use class time to engage with other people to refine their thinking and expand their understanding.

I don’t want to dull students’ creativity with homework assignments full of problems that are all similar. I want to challenge the students with an assortment of problems that require them to figure out how to apply the skills they have to solve a novel problem — just like real life. I want problems that guide students to conjecture and then prove the big results of mathematics themselves.

I want to create opportunities — through writing problem-based curricula, through experimenting with colleagues on course design, and through mentoring students in independent research — for students to engage actively in their learning of mathematics, and experience the joy of discovery and understanding.

If you would like to create a curriculum where your students discover mathematics by solving problems, and where they learn to explain their ideas to others, and create courses that suited for your particular math students, I would love to do it with you. This work is really worth doing, and I am so excited to work hard to make it happen.
References

