Definition: Let $a, b \in \mathbb{Z}$. We say that $a$ divides $b$ if there exists a $k \in \mathbb{Z}$ such that $b = ka$. Notationally, we write $a \mid b$.

It is more helpful (and generalizable) to frame divisibility in this way than the more commonplace alternative: “$a$ divides $b$ if $\frac{b}{a} \in \mathbb{Z}$.” These two definitions are almost equivalent, with the exception of the case where one considers whether $0$ divides itself. In the first definition, $0 \mid 0$; in the second, it most certainly does not.

1. Let $a, b \in \mathbb{Z}$ with $a \mid b$. Prove that for all $c \in \mathbb{Z}$, $a \mid bc$.

2. Let $a, b, c \in \mathbb{Z}$ with $a \mid b$ and $b \mid c$. Prove that $a \mid c$.

3. Let $a, b, c \in \mathbb{Z}$ with $a \mid b$ and $a \mid c$. Prove that $a \mid (\lambda b + \mu c)$ for any $\lambda, \mu \in \mathbb{Z}$.